Self-organized criticality in a neural network with the small-world topology

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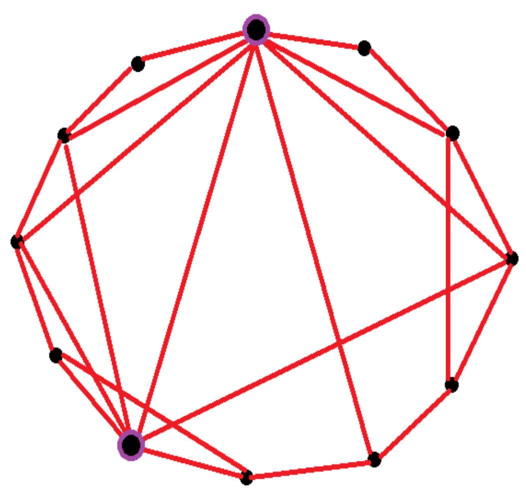
The concept of complex networks describes a set of systems that exist in nature and have non-trivial topological properties. So relationships between states, people in a group (social network (sociology)), climate networks, epydemy networks, relationships between firms, computer networks, the Web, technological networks, relationships between genes in DNA are all examples of complex networks. Various phenomena arising in such networks are of great interest. One of these interesting phenomena is the phenomenon of self-organized criticality.

Behavior in the vicinity of the point is characterized by the fact that, with a small perturbation, the system can pass the bifurcation point, thereby completely changing its behavior model.

If we imagine systems that allow us to observe the above phenomena as mathematical models, then we can see that the SOC manifests itself at a certain (critical) value of a certain parameter, called the control one. Moreover, the critical value of the control parameter may depend on some other parameters of the system.

A phenomenological Integrate-and-Fire neural network with short-term plasticity and a "Small world" topology (the Watts-Strogatz model) is taken as a model for such a system .



The state variable h>=0 is the membrane potential and is described by follows:

$$\dot{h}_i=\delta_{i\zeta(t)} I+\frac{1}{N}\sum_{j=1}^{N} u_{ij}(t_{sp}) J_{ij}(t_{sp})\,\delta\left(t-t_{sp}^{j}-\tau_d\right)$$

When *h* exceeds membrane threshold then spike happens [3]. It is then reset to the initial value. Neurons in the network are stimulated by suprathreshold current pulses *I* applied to random neuron in periodic manner. Excited neuron emits a spike and affects neurons connected to it. Stimulated neurons could emit spikes and spread the activity producing an activity avalanche. Dynamics of synaptic connections is described by follows:

$$\dot{J}_{ij}=\frac{1}{\tau_j}\left(\frac{\alpha}{u_0}-J_{ij}\right)-u_{ij}J_{ij}\,\delta\left(t-t_{sp}^{j}\right)$$

$$\dot{u}_{ij}=\frac{1}{\tau_u}\left(u_0-u_{ij}\right)+\left(1-u_{ij}\right)u_0\,\delta\left(t-t_{sp}^{j}\right)$$

where *J* – synaptic resource and *u* - utilization factor. Parameter α regulates maximum strength of connections. Coupling strength is a critical parameter for activity spreading and for SOC.

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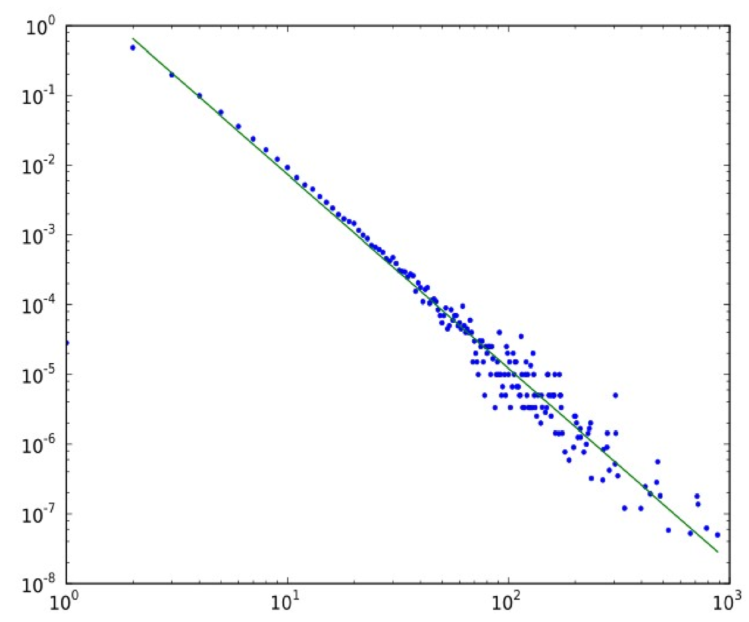
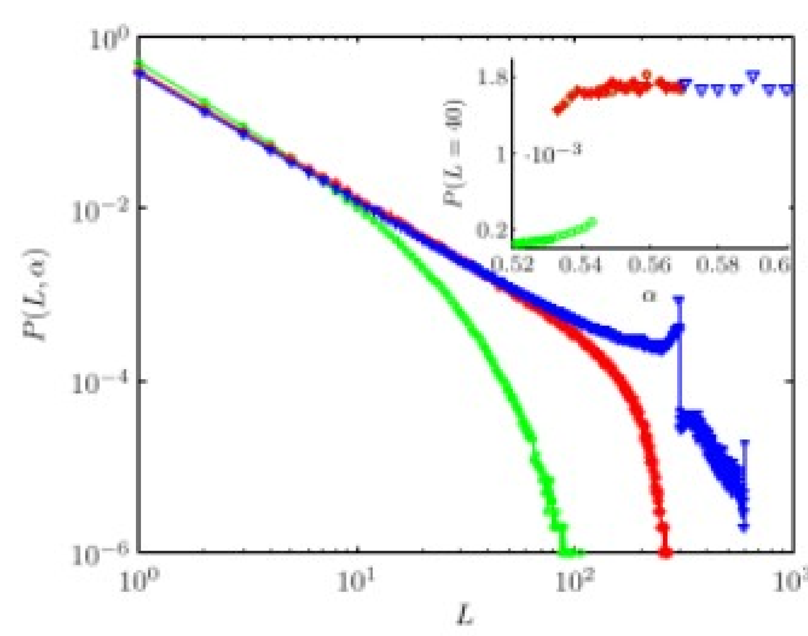
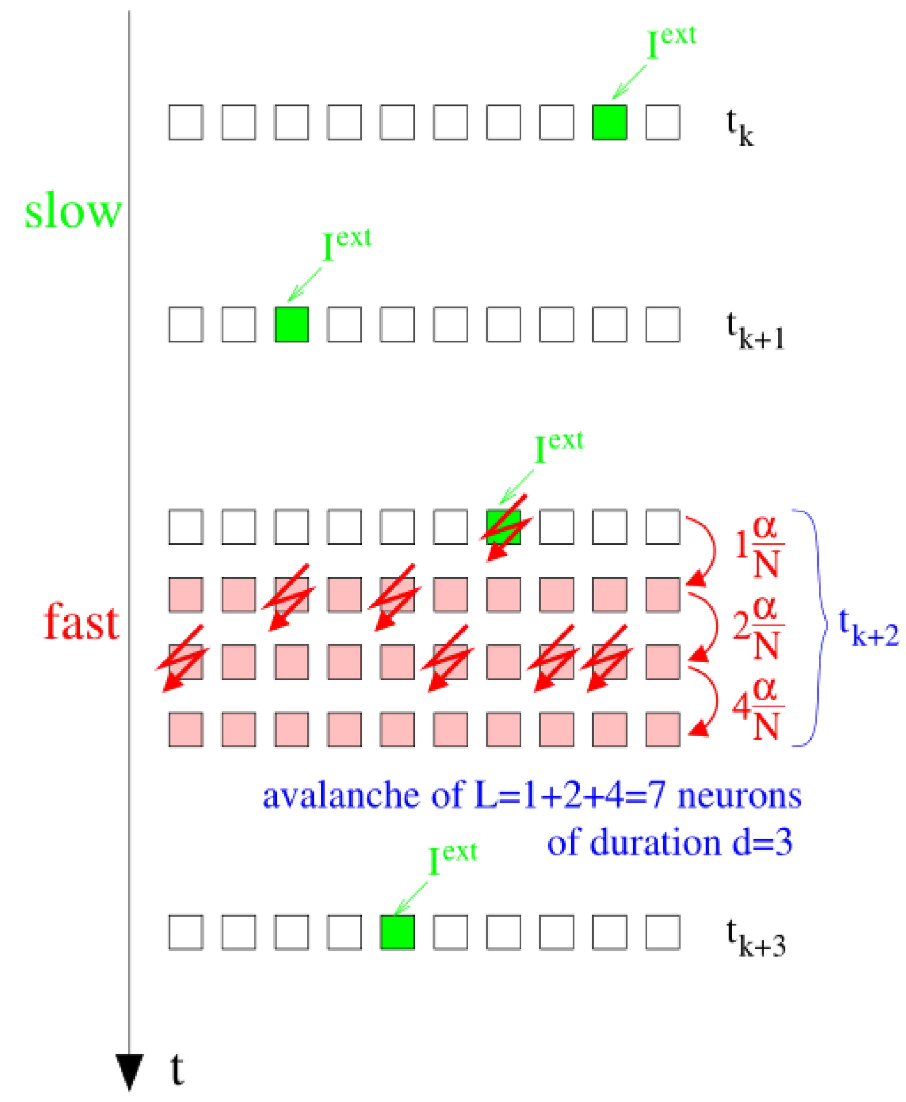
Self-organization is the process of ordering (spatial, temporal or space-time) in an open system, due to the coordinated interaction of many elements of its components. Criticality is a characteristic of a system that is on the border of a phase transition, in which a small perturbation leads to global consequences. Self-organized criticality (SOC) is a property of dynamic systems that have bifurcation points.Self-organized criticality (hereinafter referred to as SOC) describes processes, ranging from snow avalanches and earthquakes to forest fires. The dependence of the size of events of these phenomena on the number of events matches a power law.



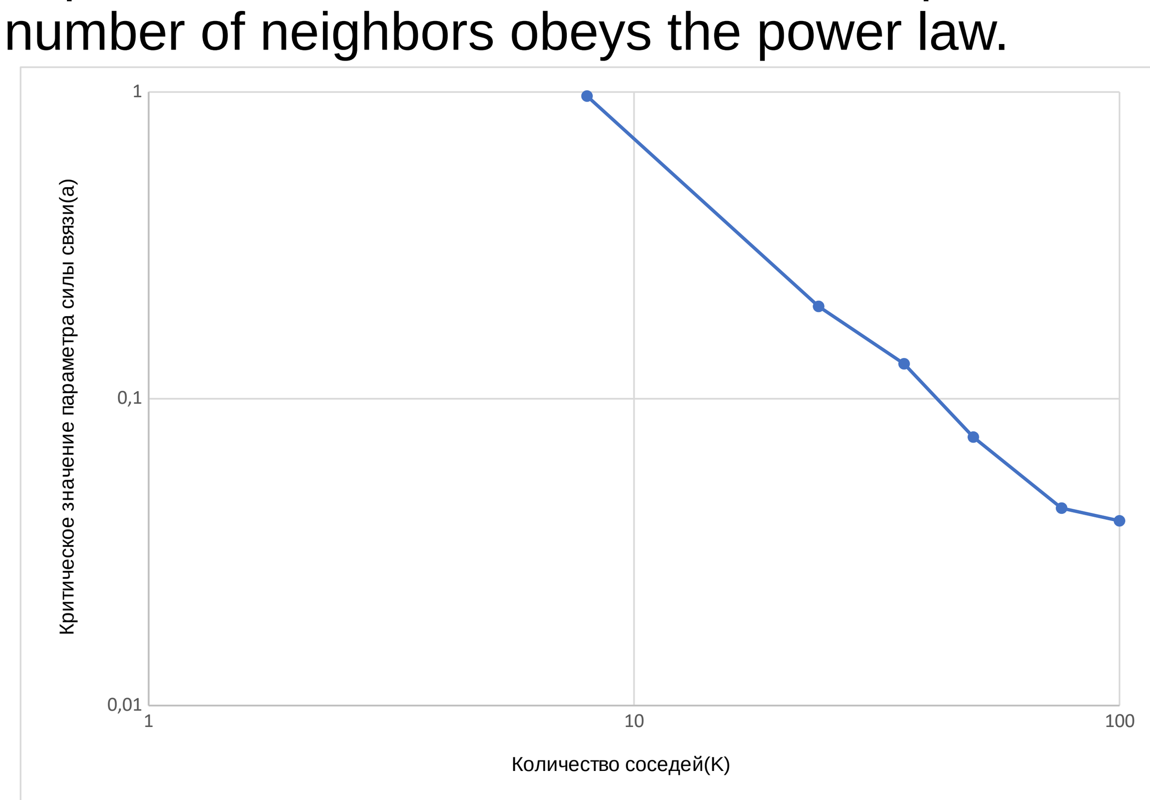


There are three dynamic network modes: subcritical(green), critical(red) and supercritical(blue).



An avalanche(event) is the activation of neurons from the moment an external stimulus is applied to the moment of no activation.
The number of activated neurons is the size of the avalanche.

The phenomenon of self-organized criticality in a neural network with short-term plasticity has been studied. It was found that this phenomenon is affected by the parameter of the bond strength. The value of the critical parameter is studied for different number of neighbors in the Watts-Strogatz topology. It seems that the dependence of the critical value of the parameter on the number of neighbors obeys the power law.



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