General characteristics of the electromagnetic fields in a resonator with hyperbolic metamaterial

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Hyperbolic metamaterials (HMM) are very promising type of the metamaterials due to their unusual properties. HMM exhibits hyperbolic-type dispersion in space of wave-vectors and described by the diagonal extremely anisotropic permittivity tensor. One of the remarkable properties of hyperbolic media (HM) is the existence of propagating waves with large wavevectors known as high-k waves in defined directions, due to the appropriate dispersion relation. This distinctive feature of the HM offers the challenge for multiple device applications and physical phenomena: negative refraction, epsilon-near-zero materials, superlenses, THz emission, nanoscale waveguiding, super-resolution imaging, quantum optics and many others. Promising properties of asymmetric hyperbolic metamaterial (AHMM) consisting of periodically arranged layers (or wires) in host media, tilted relatively to outer boundary are investigated. This kind of artificial complex material might be considered as a homogeneous medium with effective parameters due to the smallness of its period. We investigate based characteristic of electromagnetic waves propagation inside complex cavity contains AHMM by direct calculation the modes for general case of wave-vector $\mathbf{K}(k_x,k_y,k_z)$ and solve its dispersion equations numerically. We have estimated the effect of increasing of density of photonic states of electromagnetic field in the cavity, besides hyperbolic medium, by direct calculation of modal field and dispersion equation. A very high electromagnetic density of states manifests as a high rate of spontaneous emission and an enhancement of all processes of interaction of light and matter.

Challenge task:

Development of next generation photonic devices – active devices amplification and generation of the terahertz radiation or super-Planckian thermal emission

Challenge way:

New composite materials with designated properties suitable for solving of aforementioned problems - **Metamaterials**

Promise

Hyperbolic Metamaterials (HMM).

Изотропная среда

$$\mathbf{D} = \varepsilon \mathbf{E}$$

Дисперсионное ур.:

$$k_x^2 + k_y^2 + k_z^2 = \frac{\varepsilon \omega^2}{c^2}$$

Поверхность изочастот в пространстве волновых векторов:

Анизотропная среда

$$\mathbf{D} = \hat{\varepsilon} \mathbf{E}$$

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}$$

Дисперсионное ур.:

$$\frac{k_x^2}{\varepsilon_{xx}} + \frac{k_y^2}{\varepsilon_{yy}} + \frac{k_z^2}{\varepsilon_{zz}} = \frac{\omega^2}{c^2}$$

Поверхность изочастот :

Гиперболическая среда

$$\mathbf{D} = \hat{\varepsilon} \mathbf{E}$$

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{\parallel}$$

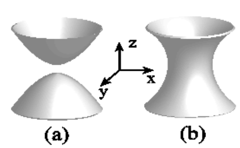
$$\varepsilon_{zz} = \varepsilon_{\perp}$$

$$\varepsilon_{\perp} \varepsilon_{\parallel} < 0$$

Дисперсионное ур.:

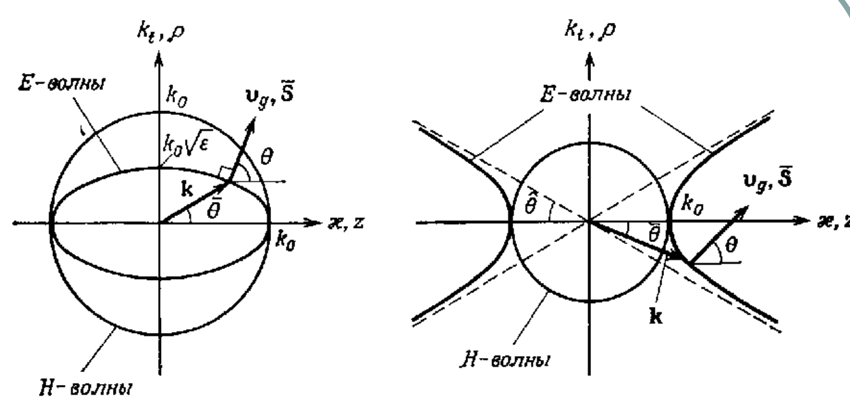
$$\frac{k_{\perp}^2}{\varepsilon_{\parallel}} - \frac{k_{\parallel}^2}{|\varepsilon_{\perp}|} = \omega^2 / c^2$$

Поверхности изочастот :



$\varepsilon_{xx} = \varepsilon_{yy} > 0$, $\varepsilon_{zz} < 0$ — $\varepsilon_{xx} = \varepsilon_{yy} < 0$, $\varepsilon_{zz} > 0$

I.V. Iorsh, I.S. Mukhin, I.V. Shadrivov, P.A. Belov, Y.S. Kivshar, " Phys. Rev. B, vol. 87, p. 075416, 2013



$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}$$

Федоров, Ф.И. *Оптика анизотропных сред* / Федоров, Ф.И. – АН БССР. Минск, 1958.

Hyperbolic media

Hyperbolic medium exhibits hyperbolic-type dispersion in space of wave-vectors and has the diagonal extremely anisotropic permittivity tensor. The dispersive properties of the hyperbolic metamaterials are inherent to uniaxial materials whose axial and tangential permittivity components are of different signs.

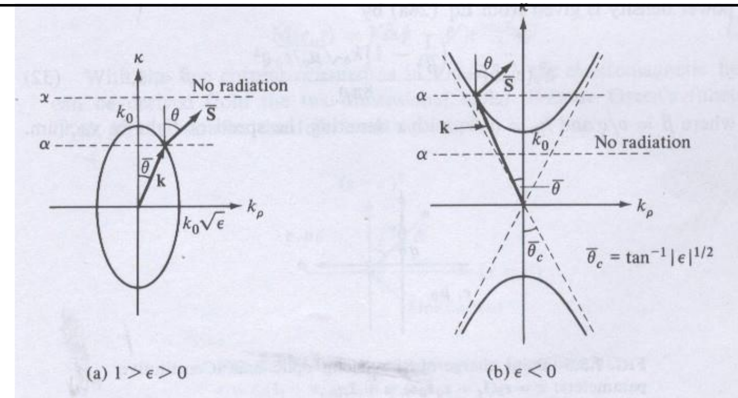
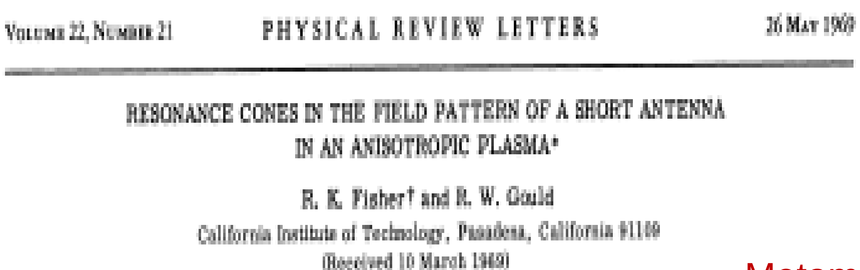


FIG. 7.3.5 Interpretation via wavenumber surfaces.

L.F. Felsen, N. Marcuvitz, *Radiation and Scattering of Waves,* 1973 (references to E. Arbel, L.B. Felsen, 1963)

Natural objects

Magnetized plasma (for RF)



VOLUME 22, NUMBER 21 PHYSICAL REVIEW LETTERS 26 MAY 1969

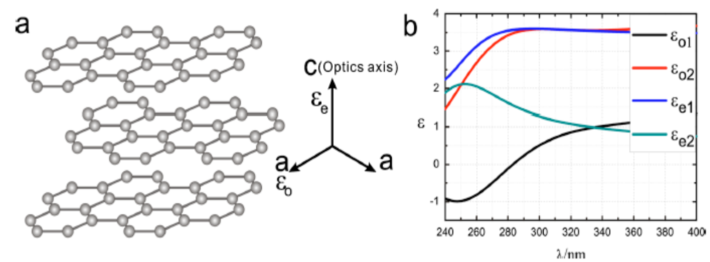
RESONANCE CONES IN THE FIELD PATTERN OF A SHORT ANTENNA IN AN ANISOTROPIC PLASMA*

R. K. Fisher† and R. W. Gould

California Institute of Technology, Pasadena, California 91109

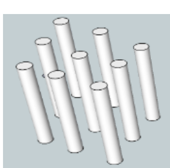
(Received 10 March 1969)

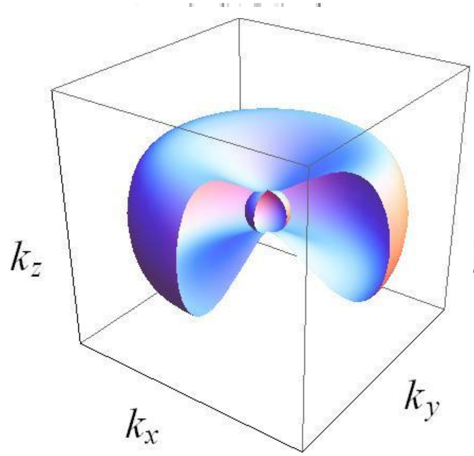
Graphite (for UV)

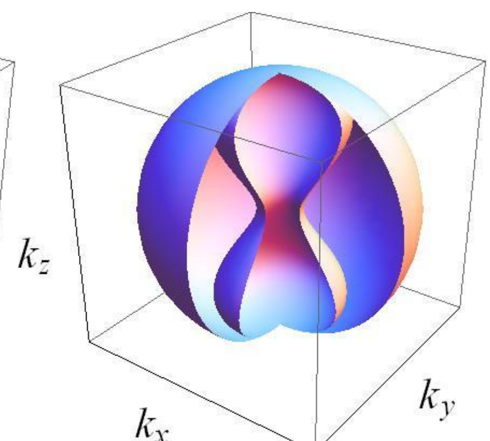


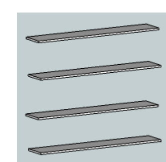
J. Sun et al. Appl. Phys. Lett. 98, *101901 (2011)*

Metamaterials







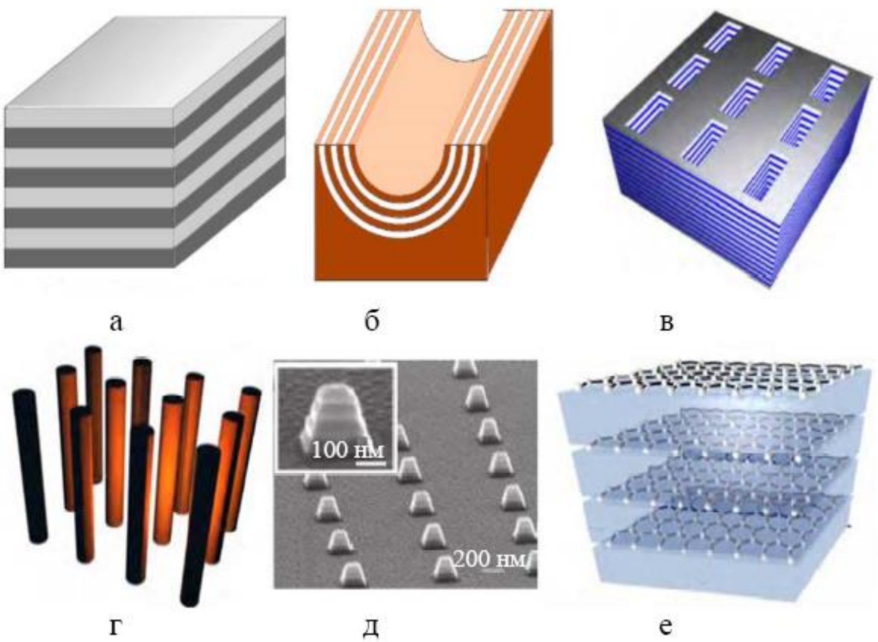


$\varepsilon_{xx} = \varepsilon_{yy} > 0$
$\varepsilon_{zz} < 0$

$\varepsilon_{xx} = \varepsilon_{yy} < 0$
$\varepsilon_{zz} > 0$

Examples of the hyperbolic metamaterials realization

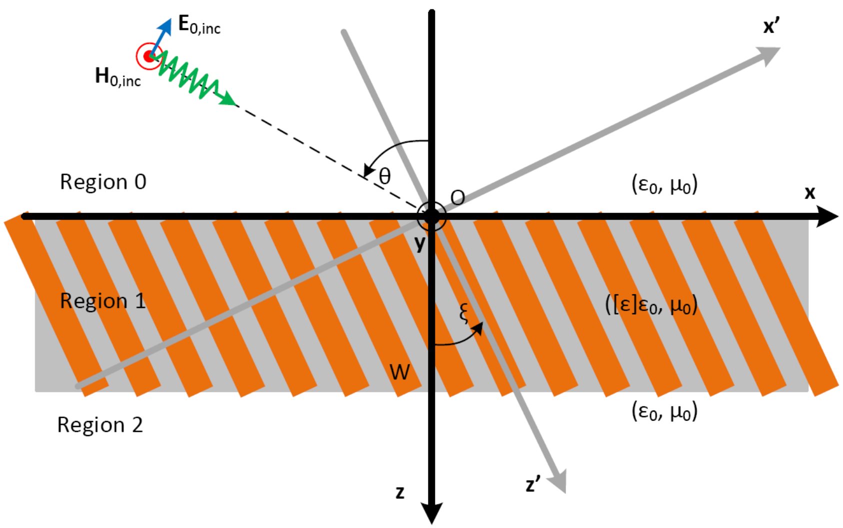
Most popular realization of HMM are metallic nanowire arrays embedded in the dielectric host matrix (г) and sub-wavelength alternating multilayer films (a).



(а) - слоистая металлодиэлектрическая структура
(б) - гиперлинзы
(в) - многослойная структура «рыболовная сеть» (fishnet)
(г) - массив нанопроводов
(д) - решетка гиперболических резонаторов, сделанных из слоистых металлодиэлектрических столбиков в наномасштабе
(е) - многослойная структура на основе графена

А.В. Щелокова, П.В. Капитанова, П.А. Белов
Методы реализации и практическое применение гиперболических метаматериалов
Научно-технический вестник информационных технологий, механики и оптики Scientific and Technical Journal of Information Technologies, Mechanics and Optics 2014, №2 (90)

Asymmetric Hyperbolic Metamaterials based on graphene levels – parameters



$$\vec{E}_0 \uparrow \qquad \sigma_s = \sigma_s(\mu_c(E_0))$$

Графен

$$\bar{\varepsilon} = \begin{bmatrix} \varepsilon_{\parallel} & 0 & 0 \\ 0 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\perp} \end{bmatrix}$$

$$\varepsilon_{\parallel} = \varepsilon_h = \varepsilon_{SiC}$$

$$\varepsilon_{\perp} = \varepsilon_{\parallel} + i\frac{\sigma(\omega)}{d\omega\varepsilon_0}$$

$$\varepsilon_{\perp} = \varepsilon_{\parallel} + \frac{i}{d\omega\varepsilon_0}\left[\frac{\sigma'(\omega, E_0)}{1+S} + i\sigma''(\omega, E_0)\right]$$

$$\sigma(\omega, E_0) = \sigma_{intra} + \sigma_{inter}$$

Г. С. Макеева, О. А. Голованов, В. В. Вареница, Д. В. Артамонов
Физико-математические науки. Физика. № 3 (31), 2014

Permittivity of SiC

$$\varepsilon_k(\omega) = \varepsilon_{k\infty} + \frac{\omega_{kp}^2}{\omega_{kTO}^2 - \omega^2 - i\gamma_k\omega}$$

$$\omega_{kp}^2 = \varepsilon_{k\infty}(\omega_{kLO}^2 - \omega_{kTO}^2)$$

$$k = p, t$$

$$\varepsilon_{SiC} = \frac{\varepsilon_{k=p} + \varepsilon_{|k=t}}{2}$$

H. Mutschke, A.C. Andersen, D. Cl.ement, Th. Henning, G. Peiter, Astron. Astrophys. 345, 187–202 (1999).

Kubo model of conductivity

$$\sigma_s(\omega, E_0) = \frac{-ie^2 k_b T}{\pi\hbar^2(\omega - i2\Gamma)}\left(\frac{\mu_c(E_0)}{k_b T} + 2\ln\left(\exp\left(\frac{-\mu_c(E_0)}{k_b T}\right)+1\right)\right) - \frac{ie^2(\omega - i2\Gamma)}{\pi\hbar^2}\int_0^{\infty}\frac{\left(\exp\left(\frac{-\xi-\mu_c(E_0)}{k_b T}\right)+1\right)^{-1} - \left(\exp\left(\frac{\xi-\mu_c(E_0)}{k_b T}\right)+1\right)^{-1}}{(\omega - i2\Gamma)^2 - \left(\frac{2\xi}{\hbar}\right)^2}d\xi$$

Hanson G. W., J. Appl. Phys. 103 064302 (2008)

Graphene gain saturation

$$\varepsilon_{\perp} = \varepsilon_{\parallel} + \frac{i}{d\omega\varepsilon_0}\left[\frac{\sigma'(\omega,\mu_c(E_0))}{(1+S)} + i\sigma''(\omega,\mu_c(E_0))\right], S = \frac{I}{I_S}.$$

$$f_d(\varepsilon) = \left(\exp\left(\frac{\varepsilon-\mu_c}{k_b T}\right)+1\right)^{-1}$$ – функция распределения Ферми – Дирака

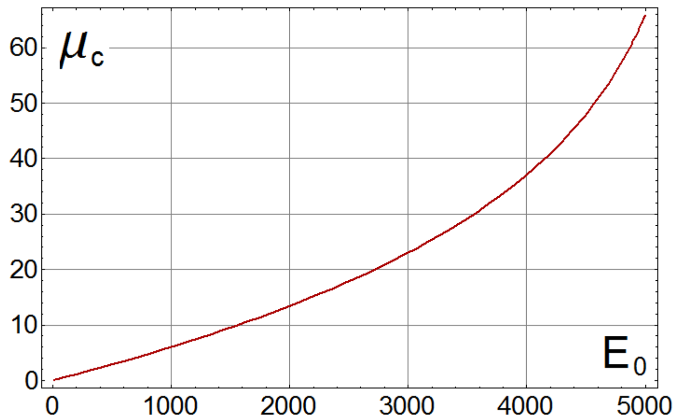
$$v_F = \frac{3\gamma_0 b}{2\hbar}$$ ($\gamma_0 = 2,7$ эВ, $b = 0,142$ нм).

ε – энергия электрона

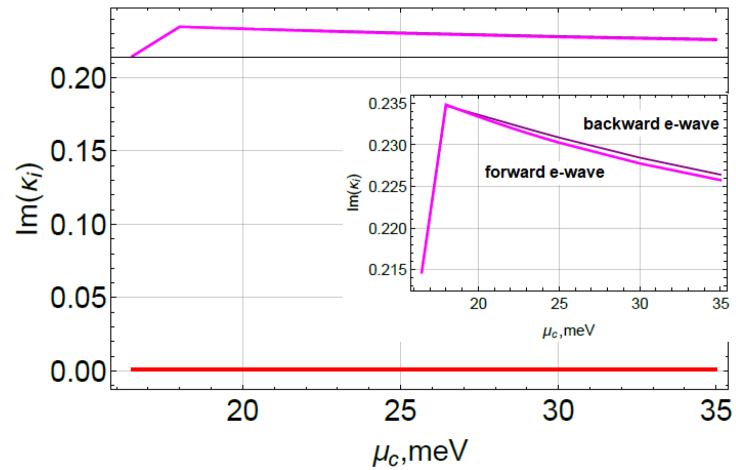
$\varepsilon_b = \varepsilon_0$

Напряженность внешнего электрического поля

$$E_0 = \frac{e}{\pi\hbar^2 v_F^2 \varepsilon_b}\int_0^{\infty}\varepsilon\left(f_d(\varepsilon) - f_d(\varepsilon+2\mu_c)\right)d\varepsilon .$$



Amplification (Im(ϰ)) vs chemical potential (meV) of grapheme



The net gain for the extraordinary waves Im($\varkappa_2$) and Im($\varkappa_3$) vs chemical potential (meV) of grapheme. Level of losses (0.0009) given by red line. $k_z = 0.07445$.

Assuming the relaxation time for charge carriers in graphene to be 10^{-12} s, we can conclude that the saturation occurs proportionally to THz field strength averaged within this temporal interval. For the frequencies about 3–4 THz the calculated value of intensity of THz radiation is about $1.2\cdot10^{15}$ W/m^2. The Poynting vector S_z value is used for the estimation of the electric field strength. We have shown that the expected THz radiation field strength which corresponds to the above saturation is about $3.5\cdot10^3$ V/nm, and $E_0 \approx 2.7\cdot10^{12}$ V/m. Of course, this value is extremely large, but it should be noted that the area of THz beam is sufficiently small and determined by the laser pump beam, hence the intensity of THz radiation can be relatively weak.

Г. С. Макеева, О. А. Голованов, В. В. Вареница, Д. В. Артамонов. // Физико-математические науки. Физика. № 3 (31), 2014

J. B. Khurgin. Graphene—A rather ordinary nonlinear optical material. Appl. Phys. Lett. 104, 161116 (2014);

S. A. Mikhailov. Comment on "Graphene—A rather ordinary nonlinear optical material. Appl. Phys. Lett. 111, 106101 (2017);

Jacob B. Khurgin. Response to "Comment on 'Graphene—A rather ordinary nonlinear optical material'" [Appl. Phys. Lett. 111, 106101 (2017)]. Appl. Phys. Lett. 111, 106102 (2017)

A.P. Mock, "Dynamic material response of graphene for FDTD simulation of passive modelocking", Proc. 13th Int. Conf. Numer. Simulation Optoelectron. Devices, pp. 71-72, Aug. 2013.

Spectral dependence of the dynamic conductivity of graphene

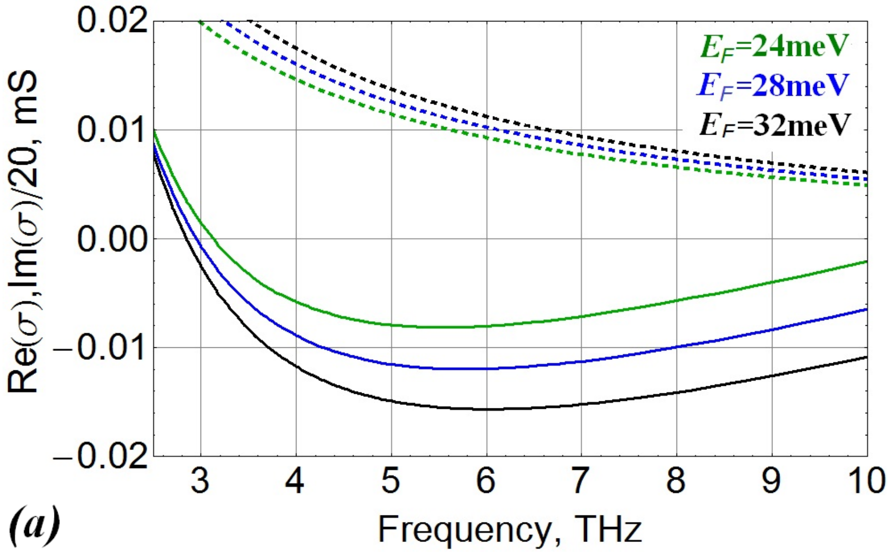
Spectral dependence of transverse permittivity of the AHMM

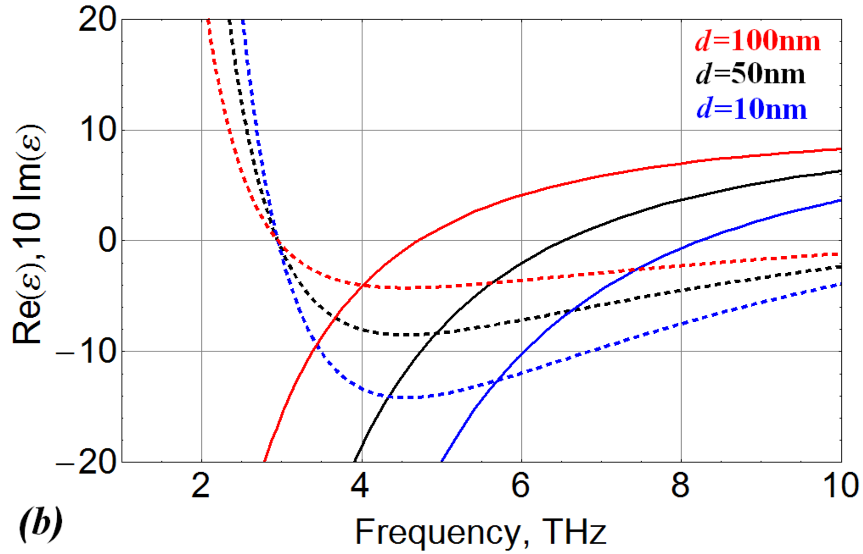
$\sigma_{gr}(\omega)$

τ=10^{-12}c
T=300ºK

E_F = 28 meV

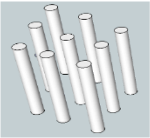
$\varepsilon_{\perp}$





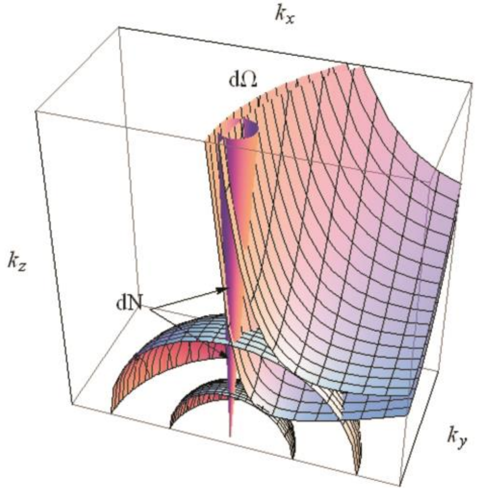
Solid lines correspond to Real part of the parameter, dashed lines correspond to Imaginary part.

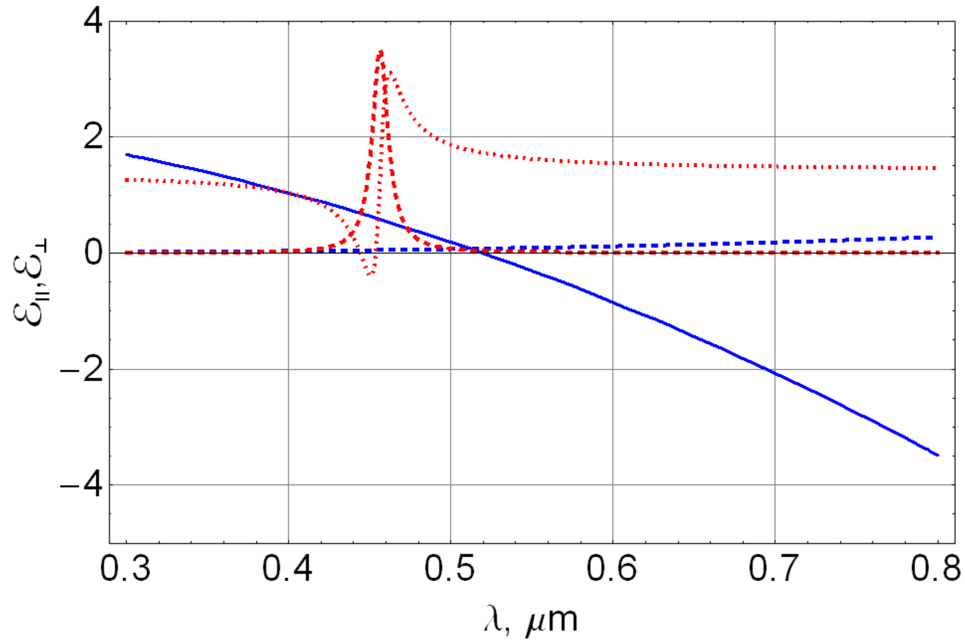
- *Козина О.Н., Мельников Л.А. Оптические характеристики асимметричного гиперболического материала. Изв. Сарат. ун-та. . Сер. Физика. 2019. Т. 19. № 2. С. 122.*
- *O.N. Kozina, L.A. Melnikov, I.S. Nefedov. Theory for terahertz laser based on a graphene hyperbolic metamaterial.* ***Journal of Optics****, 2020, V.22, No.9, P. 095003 (1-9), Q1.*
- *О. Н. Козина, Л. А. Мельников. Графеновая гиперболическая наноструктура для генерации терагерцевой волны. Радиотехника и электроника, 2022, том 67, № 10, с. 1–5.*



AHMM based on Gold Nanowires

$$\bar{\varepsilon} = \begin{bmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{bmatrix}$$





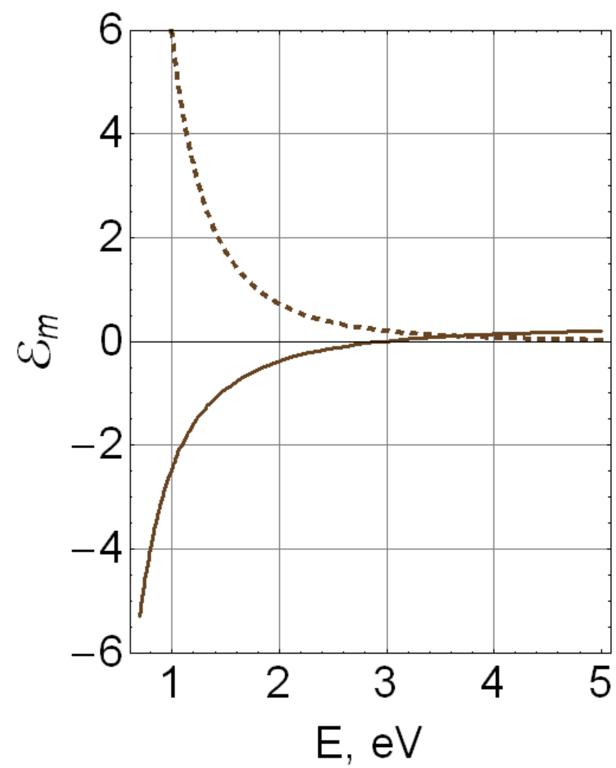
Гомогенизация проволочной среды

*C. L. Cortes, Z. Jakob, Phys. Rev. B **88**, 045407 (2013)*

$$\varepsilon_\parallel = f\varepsilon_m + (1+f)\varepsilon_d$$

$$\varepsilon_\perp = \frac{(1+f)\varepsilon_m\varepsilon_d + (1-f)\varepsilon_d^2}{(1-f)\varepsilon_m + (1+f)\varepsilon_d}$$

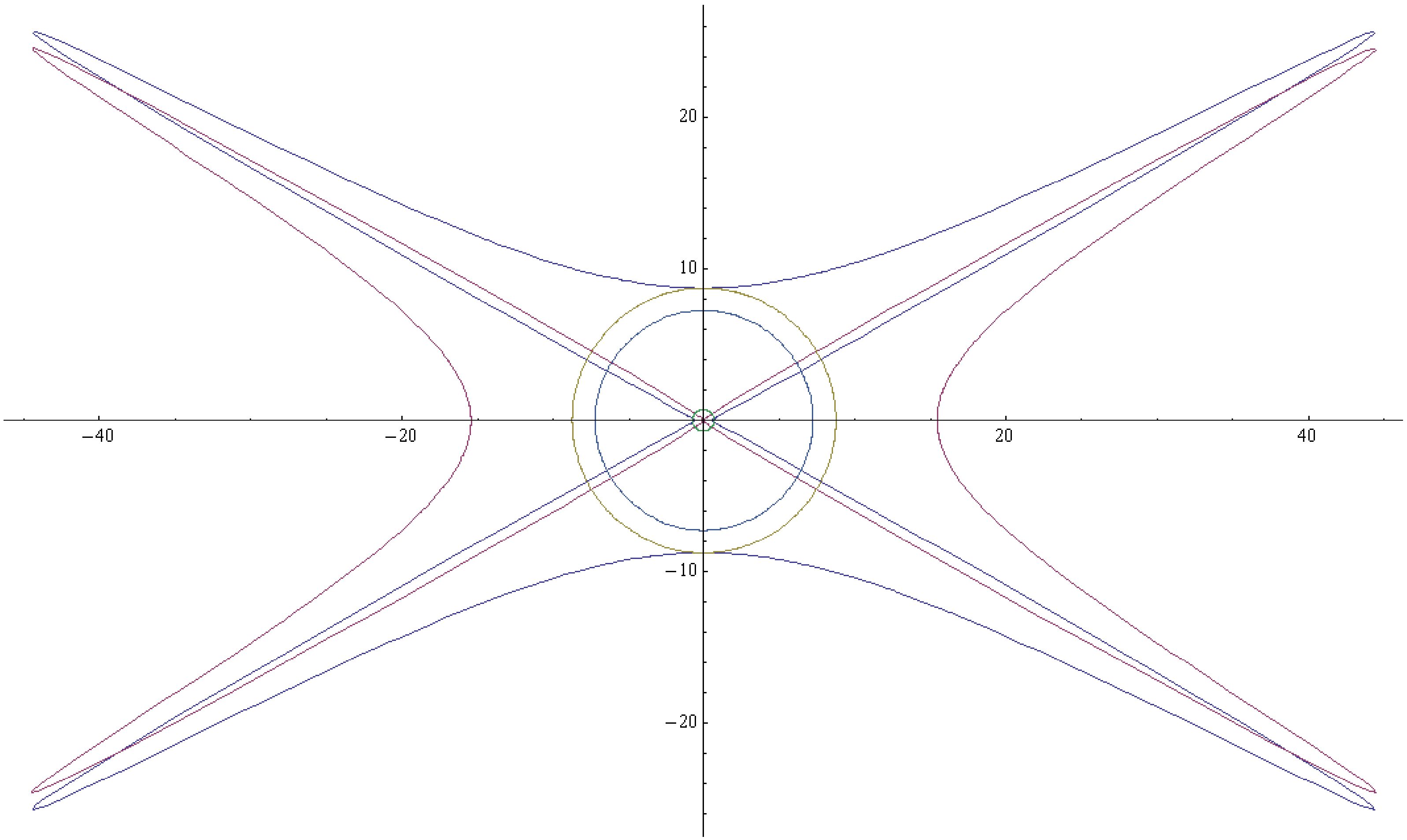
$$f = d_m/D$$



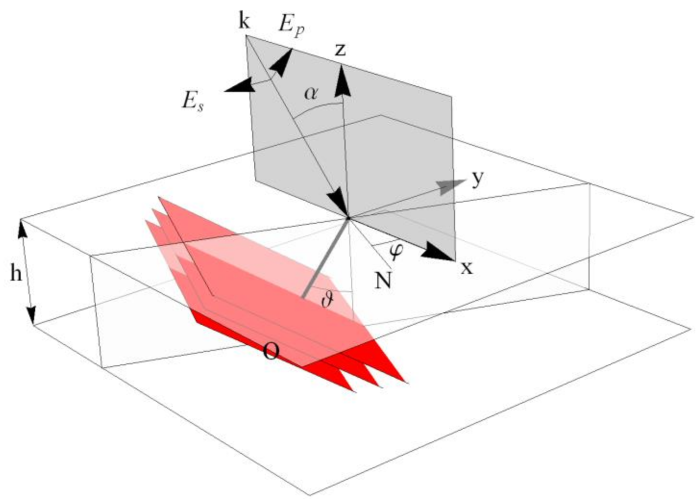
Для расчета диэлектрической функции металла ε_m используется формула Друде:

Johnson, P.B. Optical constants of the noble metals / P.B Johnson, R.W. Christy // Phys. Rev. B – 1972 – V.6 – P. 4370-4379.

Изочастоты проволочной среды на основании золота в параметрической проекции



Method of the investigation of the based characterictics of AHMM. Method Berreman 4x4 matrix.



$$\frac{\partial}{\partial z}\Psi = \frac{i\omega}{c}\Delta\Psi$$

$$\Psi = \begin{pmatrix} E_x \\ H_x \\ E_y \\ -H_y \end{pmatrix}$$

$\Psi \exp(ikr - i\omega t)$

$k=(kx,ky,kz)$

$K = \omega/c = 2\pi/\lambda$

h – thickness of AHMM

α- incidence angle

θ, φ, ψ - Euler's angles,

N - line of nodes

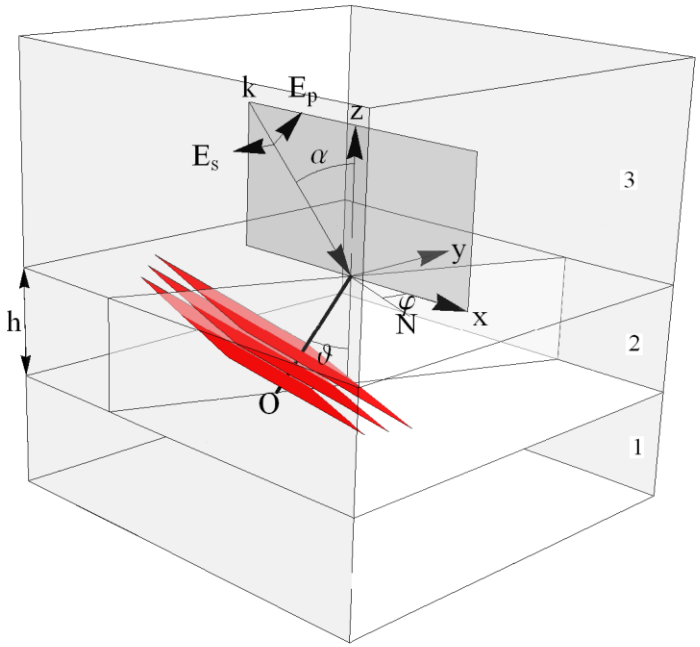
$\psi=0$

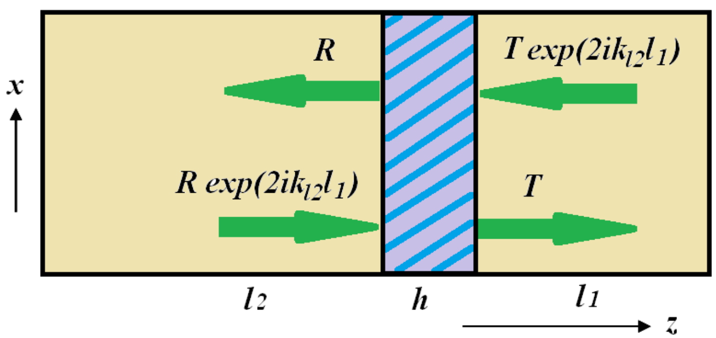
$$\Delta = \begin{pmatrix}
-\frac{k_x}{K}\frac{\varepsilon_{zx}}{\varepsilon_{zz}} & 1-\left(\frac{k_x}{K}\right)^2\frac{1}{\varepsilon_{zz}} & -\frac{k_x}{K}\frac{\varepsilon_{zy}}{\varepsilon_{zz}} & -\left(\frac{k_x}{K}\right)\left(\frac{k_y}{K}\right)\frac{1}{\varepsilon_{zz}} \\
\varepsilon_{xx}-\frac{\varepsilon_{zx}\varepsilon_{xz}}{\varepsilon_{zz}}-\left(\frac{k_y}{K}\right)^2 & -\frac{k_x}{K}\frac{\varepsilon_{xz}}{\varepsilon_{zz}} & \varepsilon_{xy}-\frac{\varepsilon_{zy}\varepsilon_{xz}}{\varepsilon_{zz}}+\left(\frac{k_x}{K}\right)\left(\frac{k_y}{K}\right) & -\frac{k_y}{K}\frac{\varepsilon_{xz}}{\varepsilon_{zz}} \\
-\frac{k_y}{K}\frac{\varepsilon_{zx}}{\varepsilon_{zz}} & -\left(\frac{k_x}{K}\right)\left(\frac{k_y}{K}\right)\frac{1}{\varepsilon_{zz}} & -\frac{k_y}{K}\frac{\varepsilon_{zy}}{\varepsilon_{zz}} & 1-\left(\frac{k_y}{K}\right)^2\frac{1}{\varepsilon_{zz}} \\
\varepsilon_{yx}-\frac{\varepsilon_{yz}\varepsilon_{zx}}{\varepsilon_{zz}}+\left(\frac{k_x}{K}\right)\left(\frac{k_y}{K}\right) & -\frac{k_x}{K}\frac{\varepsilon_{yz}}{\varepsilon_{zz}} & \varepsilon_{yy}-\frac{\varepsilon_{yz}\varepsilon_{zy}}{\varepsilon_{zz}}-\left(\frac{k_x}{K}\right)^2 & -\frac{k_y}{K}\frac{\varepsilon_{yz}}{\varepsilon_{zz}}
\end{pmatrix}$$

$\mathbf{k}=(k_x, k_y, k_z)$

Berreman D. W., Journal of the Optical Society of America, 62(4), 1157-1160 (1972).
Palto S. P., Journal of Experimental & Theoretical Physics 92(4), 552-562 (2001).
D. A. Yakovlev, V. G. Chigrinov, Modeling and optimization of the LCD optical performance (Hoi-Sing Kwok. Wiley. United Kingdom, 2015

Cavity partially filled with AHMM





Total round-trip transfer matrix

$$P_t = P_0(l_1 + l_2)P(h)$$

$\boldsymbol{P_t}$ - determine the gain and phase delay of the corresponded eigenwaves at one pass

$\boldsymbol{\Lambda_{i,k}}$=exp($i\kappa_{i,k}L$) - eigenvalues of $\boldsymbol{P_t}$

$\boldsymbol{\Lambda_{i,k}}$ - characterizes the phase delay at one pass ($L=l_1+l_2+h$).

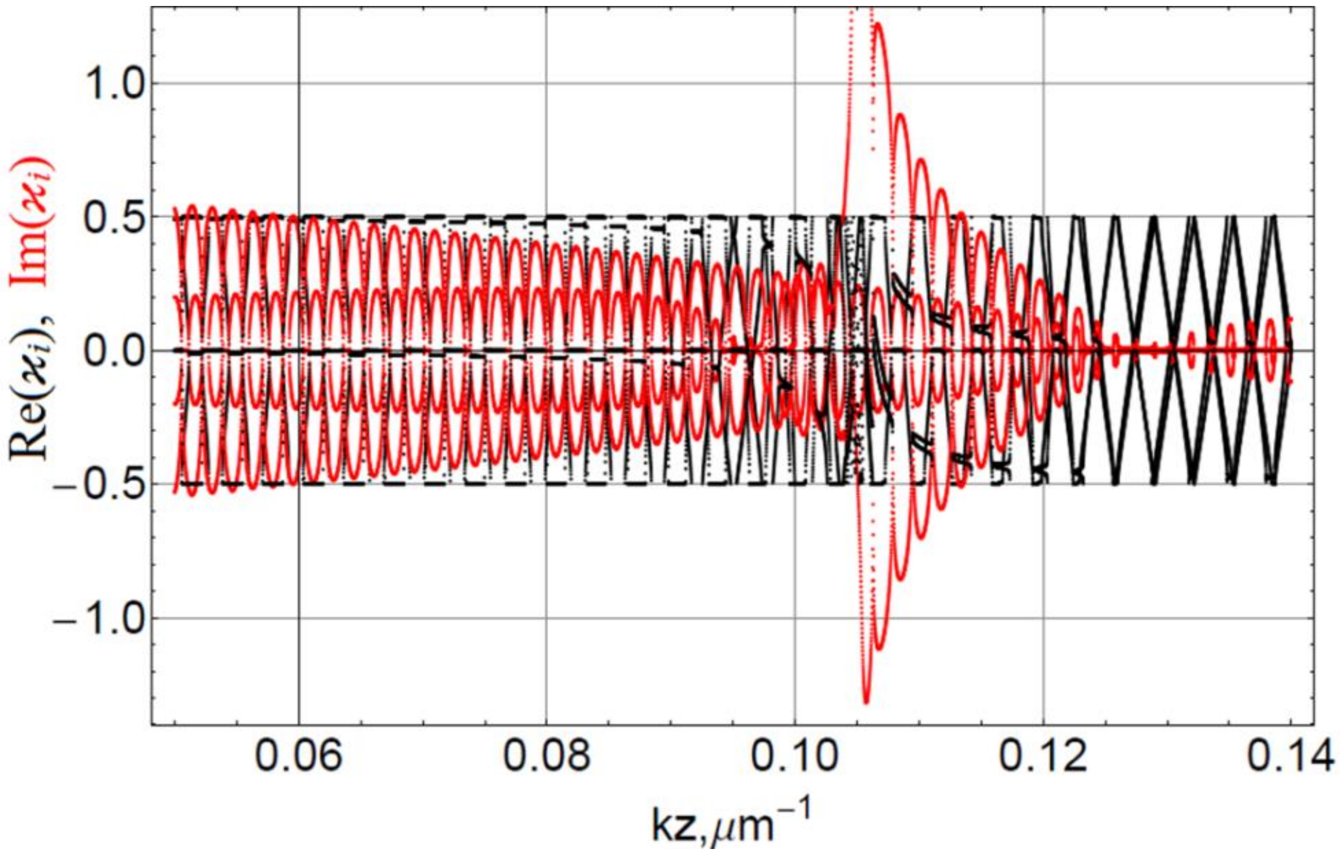
Re[$\varkappa_{i,k}$]=2πm, condition, gives the oscillation frequency of the laser

A cavity partially filled with AHM slab is modelled as an infinite number of AHM slabs, periodically placed in the isotropic lossy medium (yellow area in low sketch, 1 and 3 area in a top figure).

Eigenwaves in the cavity partially filled with AHMM based on praphene levels

$$\mathrm{Re}\left[\kappa_i(k_z,E)\right]=0, \qquad \mathrm{Im}\left[\kappa_i(k_z,E)\right]=0$$

$0.05 < k_z < 0.14$ corresponds to 2.5 до 7 ТГц



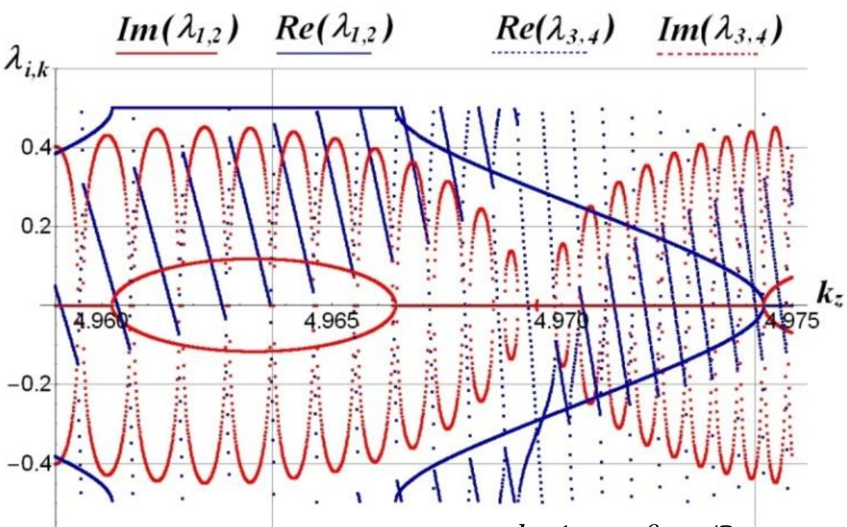
l_1 = 600 μm,
l_2 = 1320 μm
h = 5 μm
d = 50 nm

$\Lambda_i = e^{(i\varkappa_i L)}$ - eigenvalues of the entire transfer matrix P_t at a given E_0
$\varkappa_i = \ln \Lambda_i$ characterizes the phase delay at one pass ($L = l + h$)
$\mathrm{Re}(\varkappa_i) = 2\pi m$, $m = 0, \pm1, \pm2,\ldots$ determines the eigen frequencies

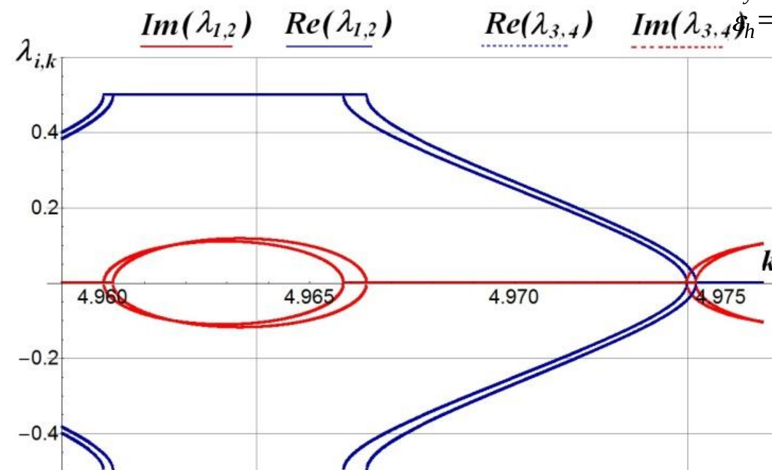
Black curves – $\mathrm{Re}(\varkappa_{i,k})$
Red curves – $\mathrm{Im}(\varkappa_{i,k})$

Eigenwaves in the cavity partially filled with AHMM based on gold wires

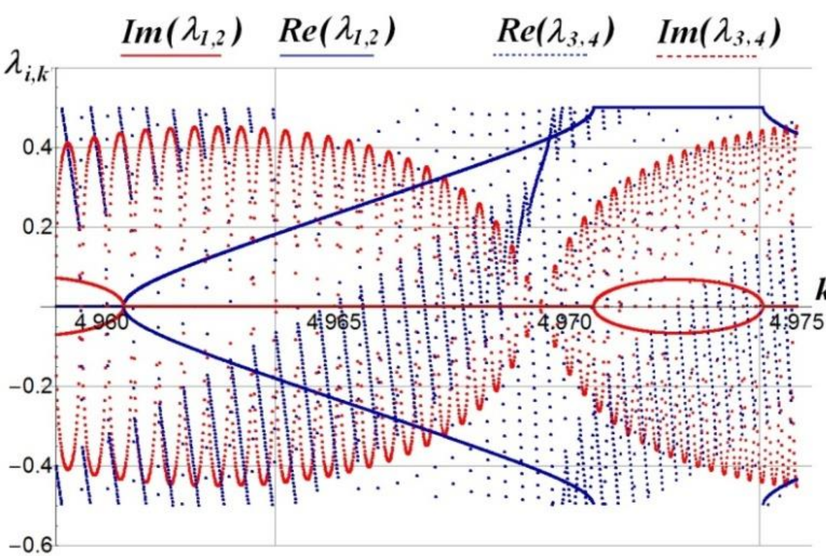
l_1=120μm
l_2=100μm
$k_x=tg(tetta)k_z$,
$k_y=0$
$\varepsilon_h=\varepsilon_{\parallel}=3$



h=1μm, $\theta \approx \pi/3$

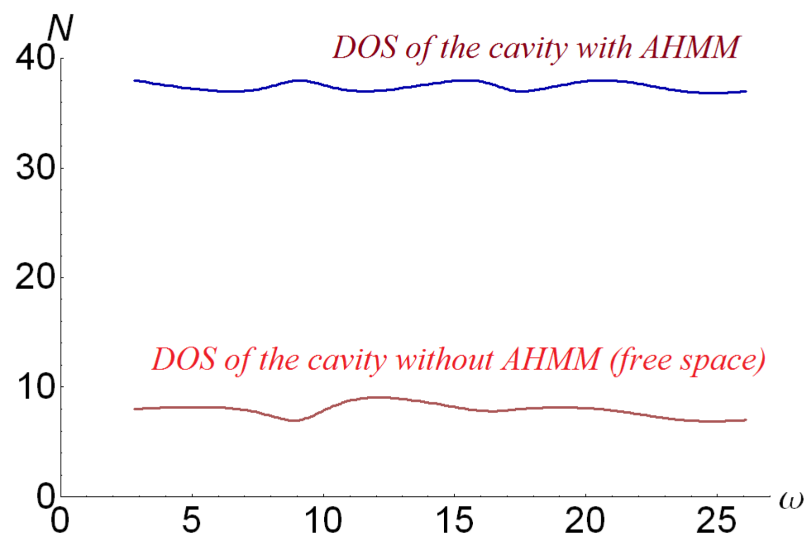


h=1μm, ϑ=0



h=2μm, $\theta \approx \pi/3$

Density of states(DOSs) in the cavity with AHMM



Conclusion

Method of the investigation asymmetrical hyperbolic medium (multilayer and nanowire) are developed.

Electromagnetic radiation in the complex cavity with anisotropic hyperbolic metamaterial are investigated using direct calculation of modal field and dispersion equation.

Based characteristic of electromagnetic waves propagation inside complex cavity contains AHMM for two cases are present.

Both forward and backward waves in the AHMM were included, this gives possibilities to include other than standing wave cavity configurations.

The eigenwaves of the cavity contains graphene AHMM at THz frequencies is calculated, accounting the saturation of the gain. The frequency of oscillation and field intensity was calculated from the solution of equations for real and imaginary parts of log of eigenvalues of total transfer matrix of one period of the structure.

Effect of increasing of density of photonic states of electromagnetic field in the cavity, besides hyperbolic medium contains golden nanowires was estimated.

A very high electromagnetic density of states manifests as a high rate of spontaneous emission and an enhancement of all processes of interaction of light and matter.