Terahertz laser based on a hyperbolic metamaterial consisting of thin layers of graphene

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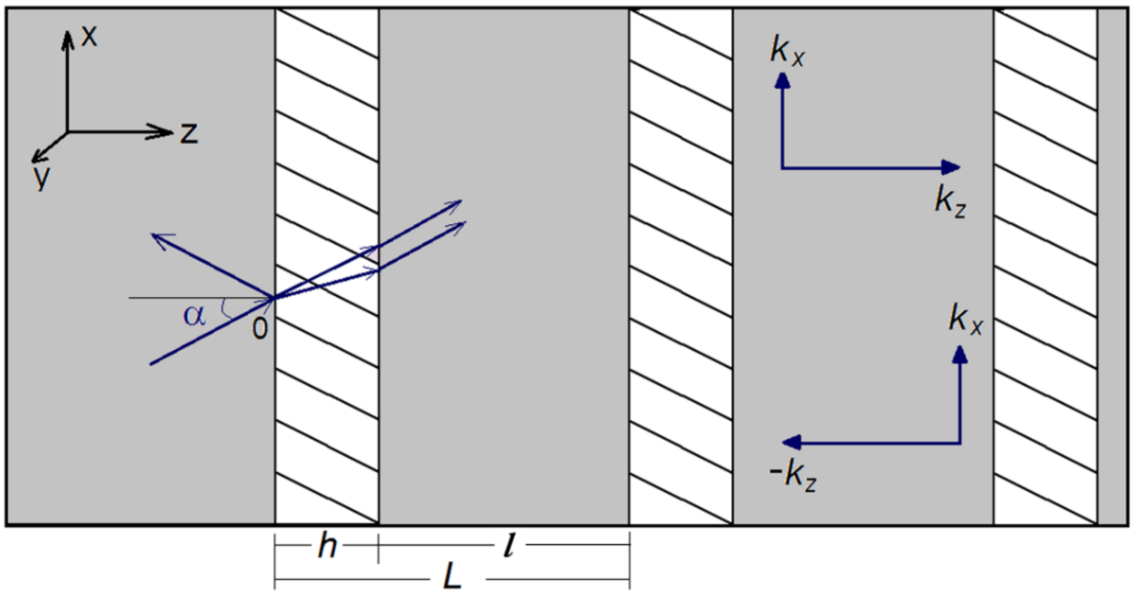
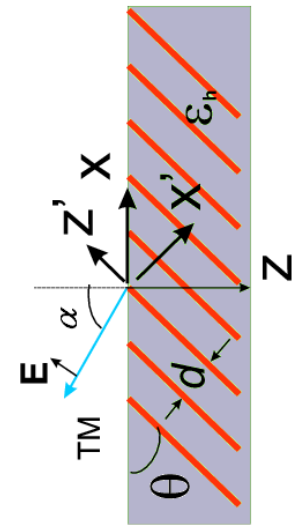
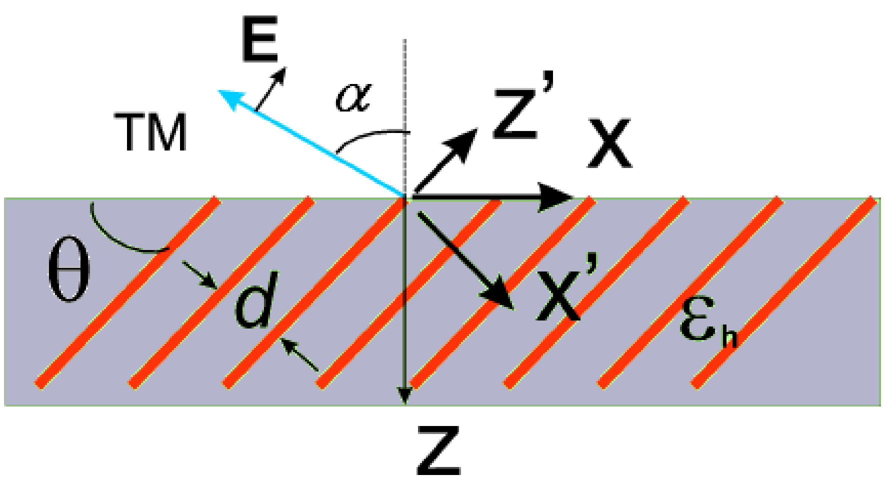
Theoretical investigation of the terahertz lasing in the cavity containing graphene-based asymmetric hyperbolic metamaterial was present. The metamaterial under investigation is a nanoscale structure composed of periodically arranged layers of a semiconductor and inverted graphene. The aim of study is development of the component base for creating devices for terahertz generation and manipulation. The radiation characteristics in the cavity have been investigated by the transfer matrix method. A hyperbolic metamaterial is considered as a homogeneous medium with effective parameters due to the smallness of its period. The optimal conditions and parameters of the structure for an effective terahertz generation have been determined by numerical simulation. Generation linewidth, graphene saturation intensity, radiation power and the impact of the geometric parameters of structure on the process of terahertz wave generation have been estimated.

Cavity containing graphene-based asymmetric hyperbolic metamaterial.

Complex resonator with thin hyperbolic media inside.



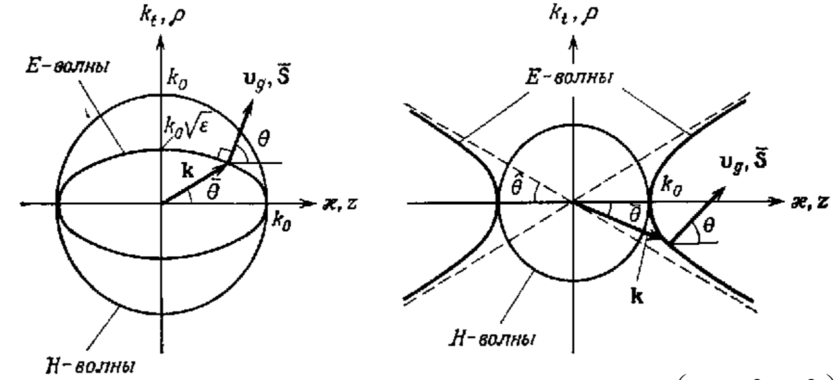




Hyperbolic media is a nanoscale metamaterial composed of periodically arranged layers of a semiconductor (gray areas) and inverted graphene (red lines) with optics axis tilted with respect to outer boundary as an active media

Hyperbolic media

Hyperbolic medium exhibits hyperbolic-type dispersion in space of wave-vectors and has the diagonal extremely anisotropic permittivity tensor. The dispersive properties of the hyperbolic metamaterials are inherent to uniaxial materials whose axial and tangential permittivity components are of different signs.

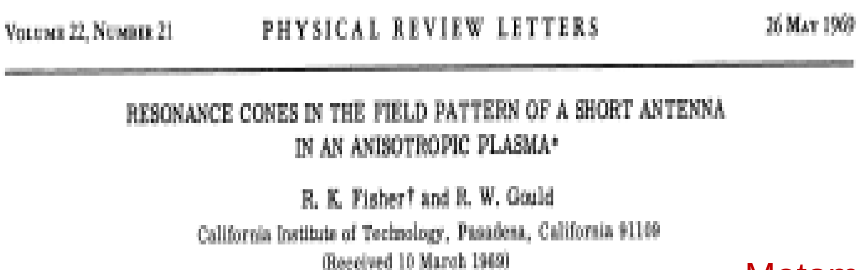


Федоров, Ф.И. *Оптика анизотропных сред* / Федоров, Ф.И. – АН БССР. Минск, 1958.

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}$$

Natural objects

Magnetized plasma (for RF)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RESONANCE CONES IN THE FIELD PATTERN OF A SHORT ANTENNA IN AN ANISOTROPIC PLASMA*

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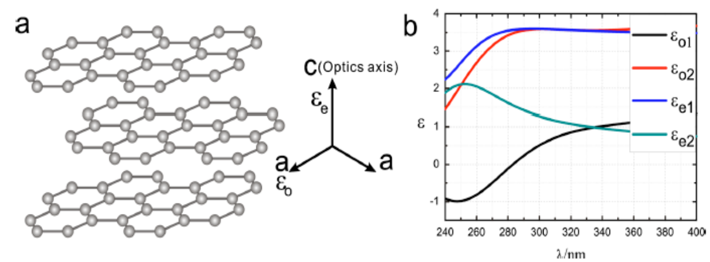
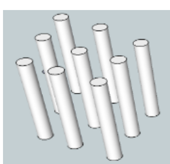
(Received 10 March 1969)

Graphite (for UV)

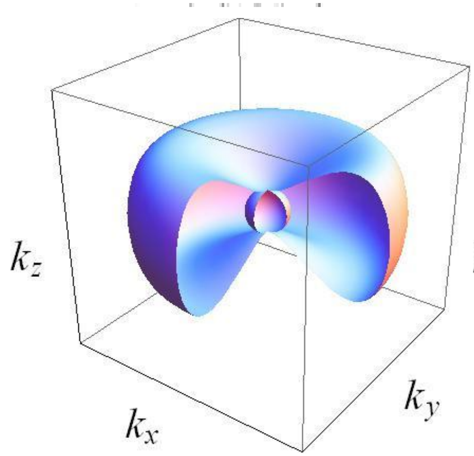


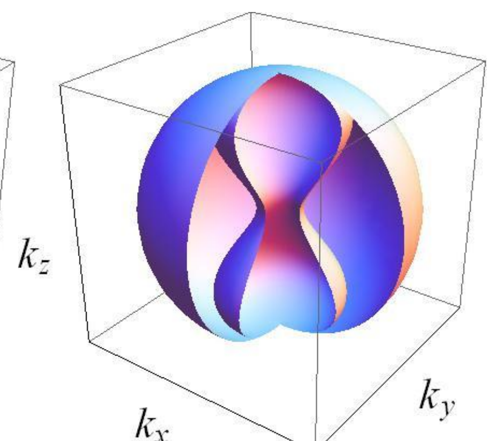
J. Sun et al. Appl. Phys. Lett. 98, *101901 (2011)*

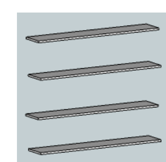
Metamaterials



$\varepsilon_{xx} = \varepsilon_{yy} > 0$
$\varepsilon_{zz} < 0$

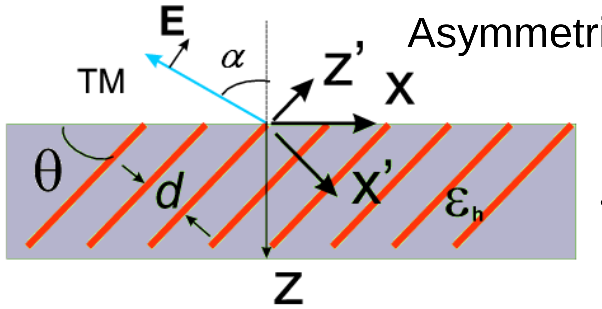






$\varepsilon_{xx} = \varepsilon_{yy} < 0$
$\varepsilon_{zz} > 0$

Asymmetric Hyperbolic Metamaterials – parameters



$$\bar{\varepsilon} = \begin{bmatrix} \varepsilon_{\parallel} & 0 & 0 \\ 0 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\perp} \end{bmatrix}$$

$$\varepsilon_{\perp} = \varepsilon_{\parallel} + i\frac{\sigma(\omega)}{d\omega\varepsilon_0} \qquad \varepsilon_{\parallel} = \varepsilon_h = \varepsilon_{SiC}$$

Kubo model of graphen conductivity

$$\sigma_s(\omega, E_0) = \frac{-ie^2 k_b T}{\pi\hbar^2(\omega - i2\Gamma)}\left(\frac{\mu_c(E_0)}{k_b T} + 2\ln\left(\exp\left(\frac{-\mu_c(E_0)}{k_b T}\right)+1\right)\right) - \frac{ie^2(\omega - i2\Gamma)}{\pi\hbar^2}\int_0^{\infty}\frac{\left(\exp\left(\frac{-\xi-\mu_c(E_0)}{k_b T}\right)+1\right)^{-1} - \left(\exp\left(\frac{\xi-\mu_c(E_0)}{k_b T}\right)+1\right)^{-1}}{(\omega - i2\Gamma)^2 - \left(\frac{2\xi}{\hbar}\right)^2}d\xi$$

Permittity of SiC

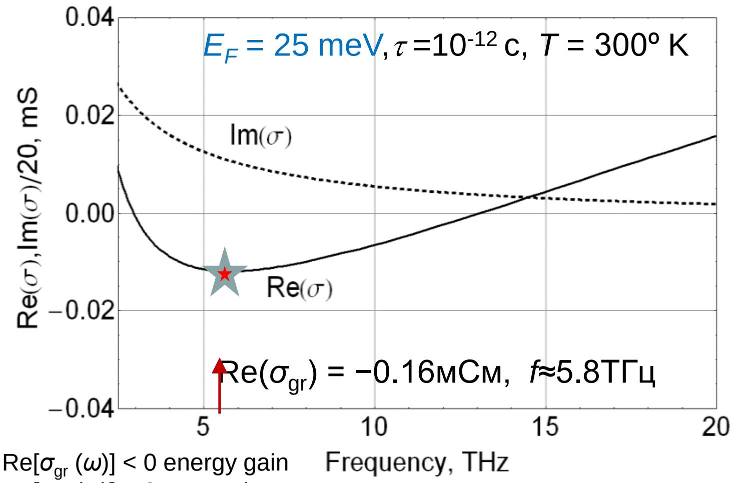
$$\varepsilon_k(\omega) = \varepsilon_{k\infty} + \frac{\omega_{kp}^2}{\omega_{kTO}^2 - \omega^2 - i\gamma_k\omega}$$

$$\omega_{kp}^2 = \varepsilon_{k\infty}(\omega_{kLO}^2 - \omega_{kTO}^2)$$

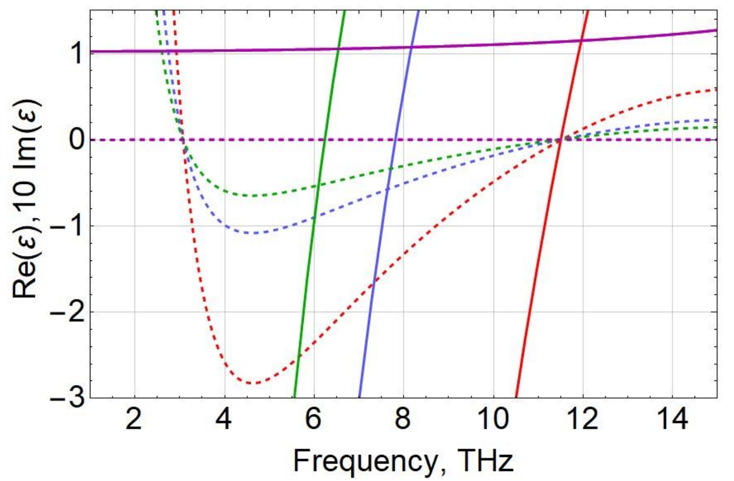
$$k = p, t$$

$$\varepsilon_{SiC} = \frac{\varepsilon_{k=p} + \varepsilon_{|k=t}}{2}$$

H. Mutschke, A.C. Andersen, D. Cl.ement, Th. Henning, G. Peiter, Astron. Astrophys. 345, 187–202 (1999).



Re[σ_{gr} (ω)] < 0 energy gain
Re[σ_{gr} (ω)] > 0 energy loss



Re($\varepsilon_{\perp}$)<0 - hyperbolic properties
Im($\varepsilon_{\perp}$) characterizes the gain properties of HMM:
Im($\varepsilon_{\perp}$)<0 - amplification.

Frequency region from 2.5 to 7 THz is suitable for THz amplification

- Козина О.Н., Мельников Л.А. Оптические характеристики асимметричного гиперболического материала. Изв. Сарат. ун-та. . Сер. Физика. 2019. Т. 19. № 2. С. 122.
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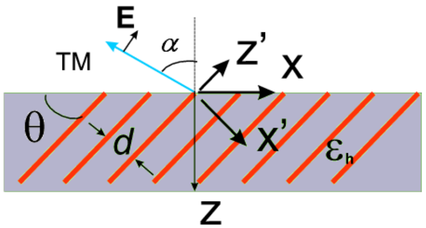
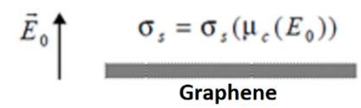
- О. Н. Козина, Л. А. Мельников. Графеновая гиперболическая наноструктура для генерации терагерцевой волны. Радиотехника и электроника, 2022, том 67, № 10, с. 1–5.

Asymmetric Hyperbolic Metamaterials – parameters with taking into account the saturation of amplification in graphene



$$\varepsilon_{\perp} = \varepsilon_{\parallel} + \frac{i}{d\omega\varepsilon_0}\left[\frac{\sigma'(\omega, E_0)}{1+S} + i\sigma''(\omega, E_0)\right]$$

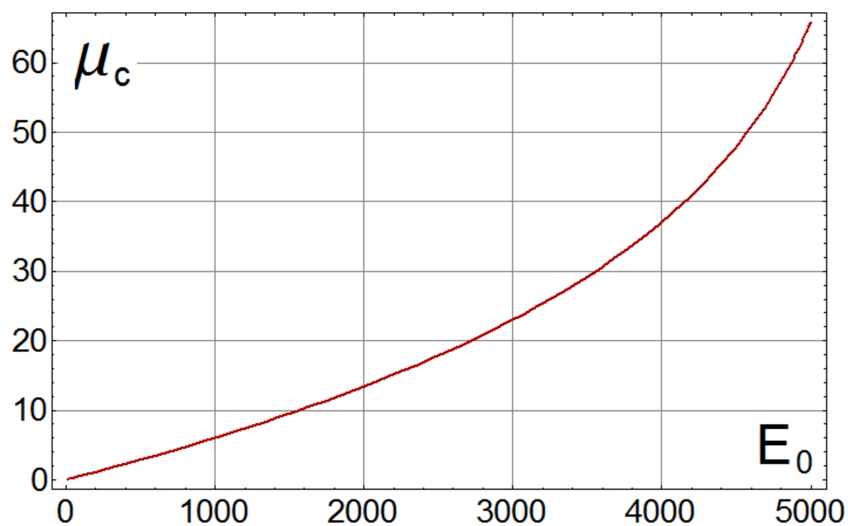


The dependence of chemical potential (meV) of graphene from transverse electric field (V/nm).
E_0 is a component of the external electric strength vector transverse to the graphene plane

$$E_0 = \frac{e}{\pi\hbar^2 v_F^2 \varepsilon_b}\int_0^{\infty} d\varepsilon\left(f_d(\varepsilon) - f_d(\varepsilon + 2\mu_c)\right)$$

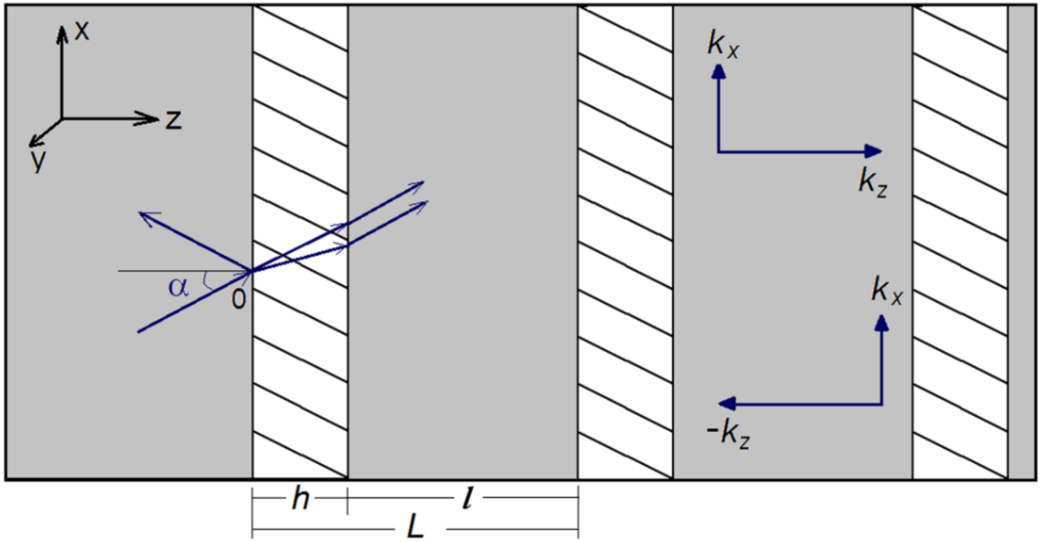
$$f_d(\varepsilon) = 1/(\exp[\frac{\varepsilon - \mu_c}{k_B T}] + 1)$$

$f_d(\varepsilon)$ – the Fermi-Dirac function
μ_c – the chemical potential
ε_b – the dielectric constant of graphene



Г. С. Макеева, О. А. Голованов, В. В. Вареница, Д. В. Артамонов. // Физико-математические науки. Физика. № 3 (31), 2014
J. B. Khurgin. Graphene—A rather ordinary nonlinear optical material. Appl. Phys. Lett. 104, 161116 (2014);

Cavity partially filled with AHMM



Total round-trip transfer matrix

$$P_t = P_0(l)P(h)$$

$$\Psi_{\mathrm{T}} = \mathbf{P}(h)(\Psi_{\mathrm{I}} + \Psi_{\mathrm{R}})$$

$$\Delta_{i,j} = f(\varepsilon_{\perp}, \varepsilon_{||}, \theta, \varphi, \psi, k_x), \quad i, j = 1,2,3,4$$

Due to the linearity of the eigenvalues problem at a given E_0 the eigenvalues of the entire transfer matrix P_t are

$$\Lambda_i = e^{(i\varkappa_i L)},$$

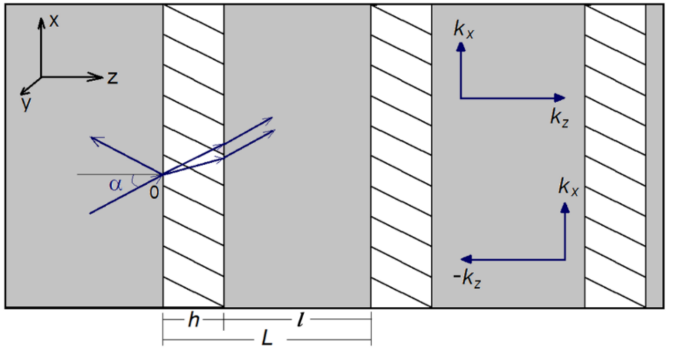
where $\varkappa_i = \ln \Lambda_i$ characterizes the phase delay at one pass ($L = l + h$).

Mode frequency and mode intensity proportional to E_0 are the solutions of the equations:

$$\mathrm{Re}\left[\kappa_i(k_z, E)\right] = 0, \qquad \mathrm{Im}\left[\kappa_i(k_z, E)\right] = 0$$

These equations may be solved numerically to find k_{z0} and E_0.
Due to a weak dependence of k_{z0} from E_0, k_{z0} can be easily derived from the first equation, and E_0 from the second equation.

Eigenwaves in the cavity partially filled with AHMM

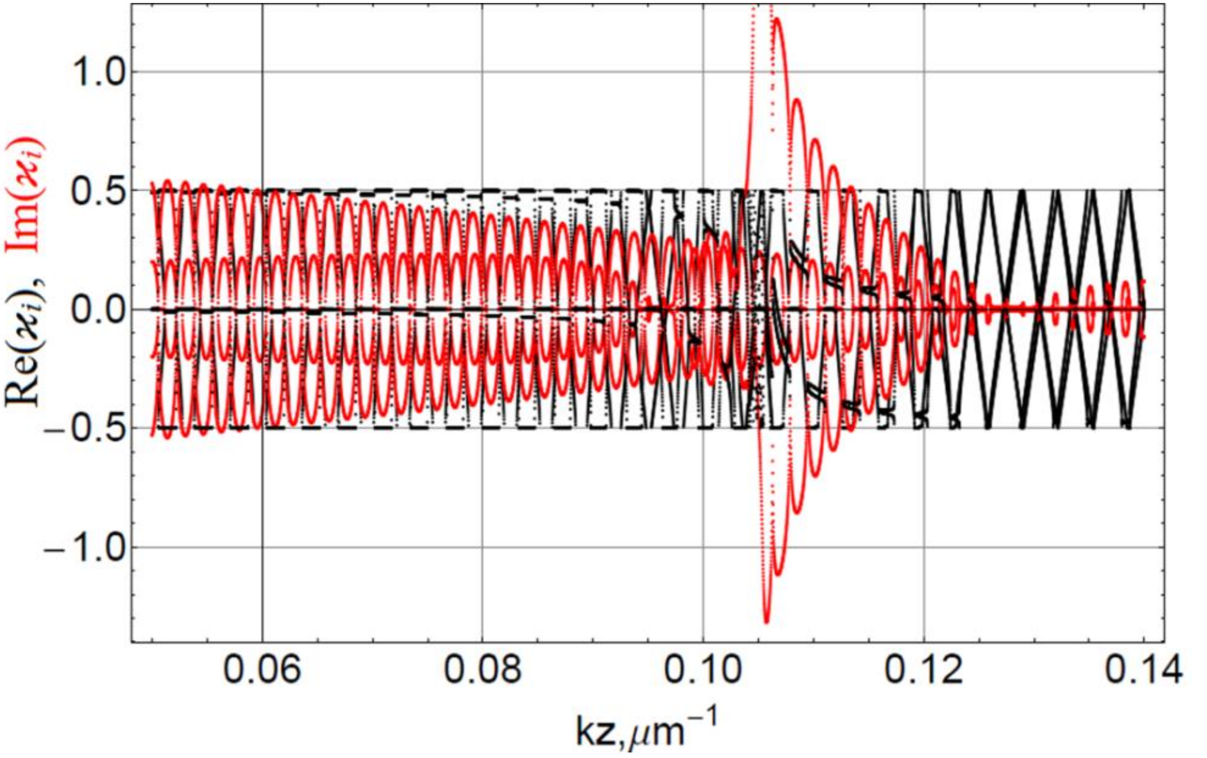


$$\mathrm{Re}\left[\kappa_i(k_z, E)\right] = 0, \qquad \mathrm{Im}\left[\kappa_i(k_z, E)\right] = 0$$

l_1 = 600 μm,
l_2 = 1320 μm
h = 5 μm
d = 50 nm

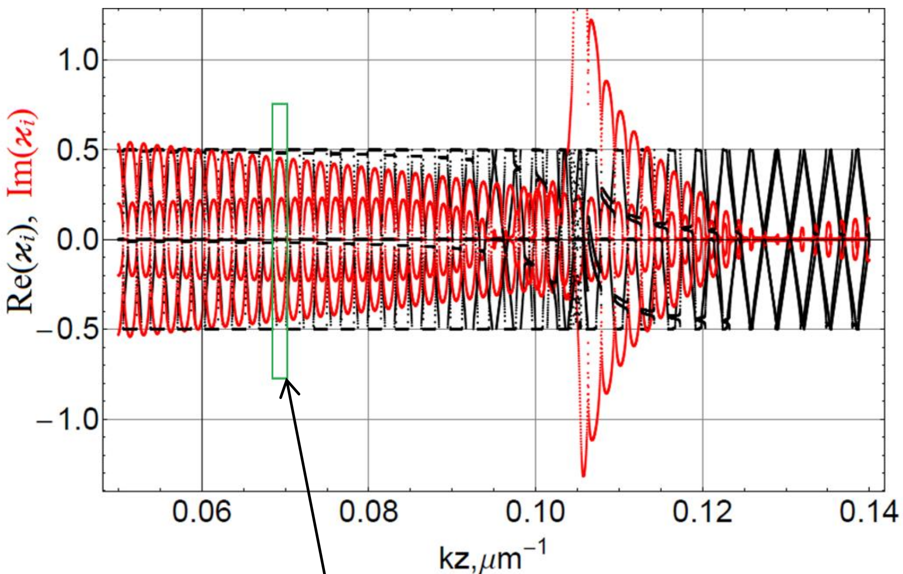
Black curves – Re($\varkappa_{i,k}$)
Red curves – Im($\varkappa_{i,k}$)

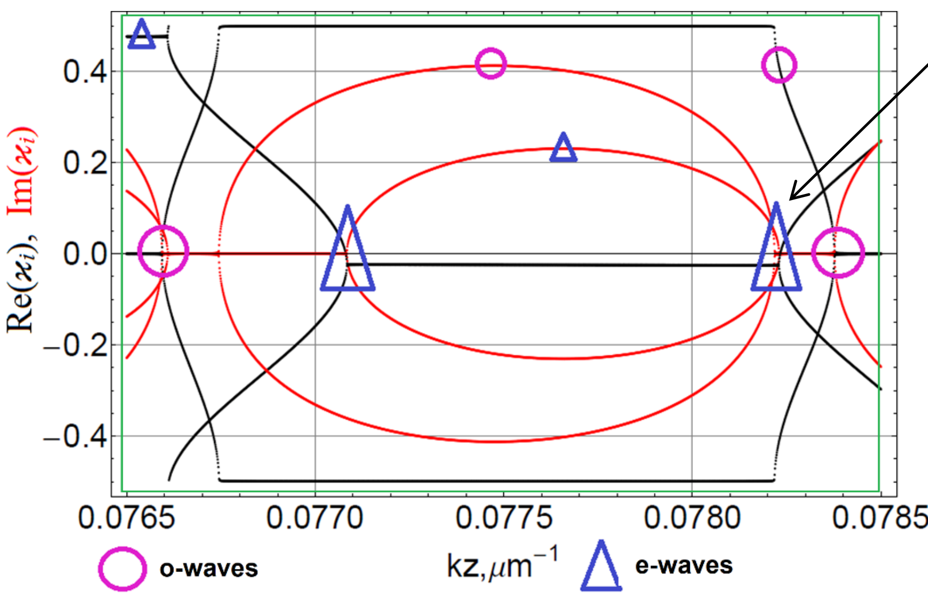
0.05 < k_z < 0.14 corresponds to 2.5 до 7 ТГц



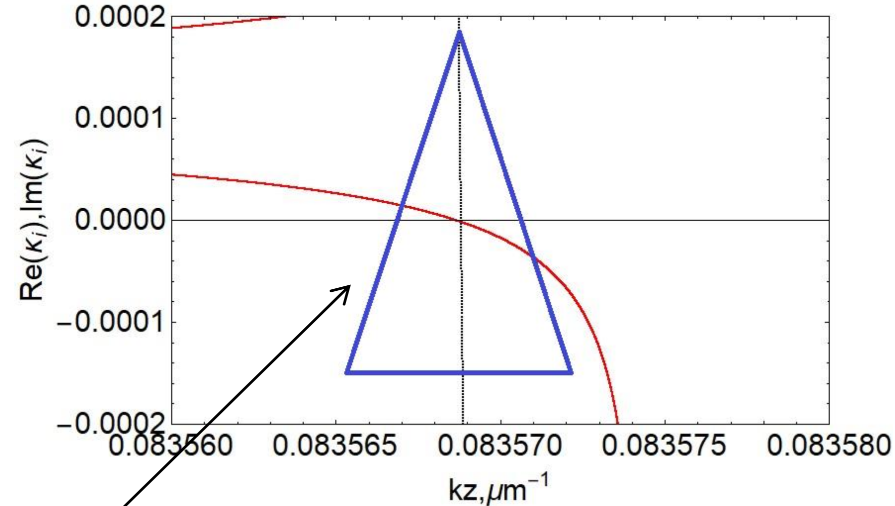
$\Lambda_i = e^{(i\varkappa_i L)}$ - eigenvalues of the entire transfer matrix P_t at a given E_0
$\varkappa_i = \ln \Lambda_i$ characterizes the phase delay at one pass ($L = l + h$)
Re($\varkappa_i$) = $2\pi m$, m = 0, ±1, ±2,… determines the eigen frequencies

Eigenvalues $\varkappa_{i,k}$ vs k_z





Mode generation condition determined by the equation
$\mathrm{Re}[\varkappa_i(k_z)] = 0$



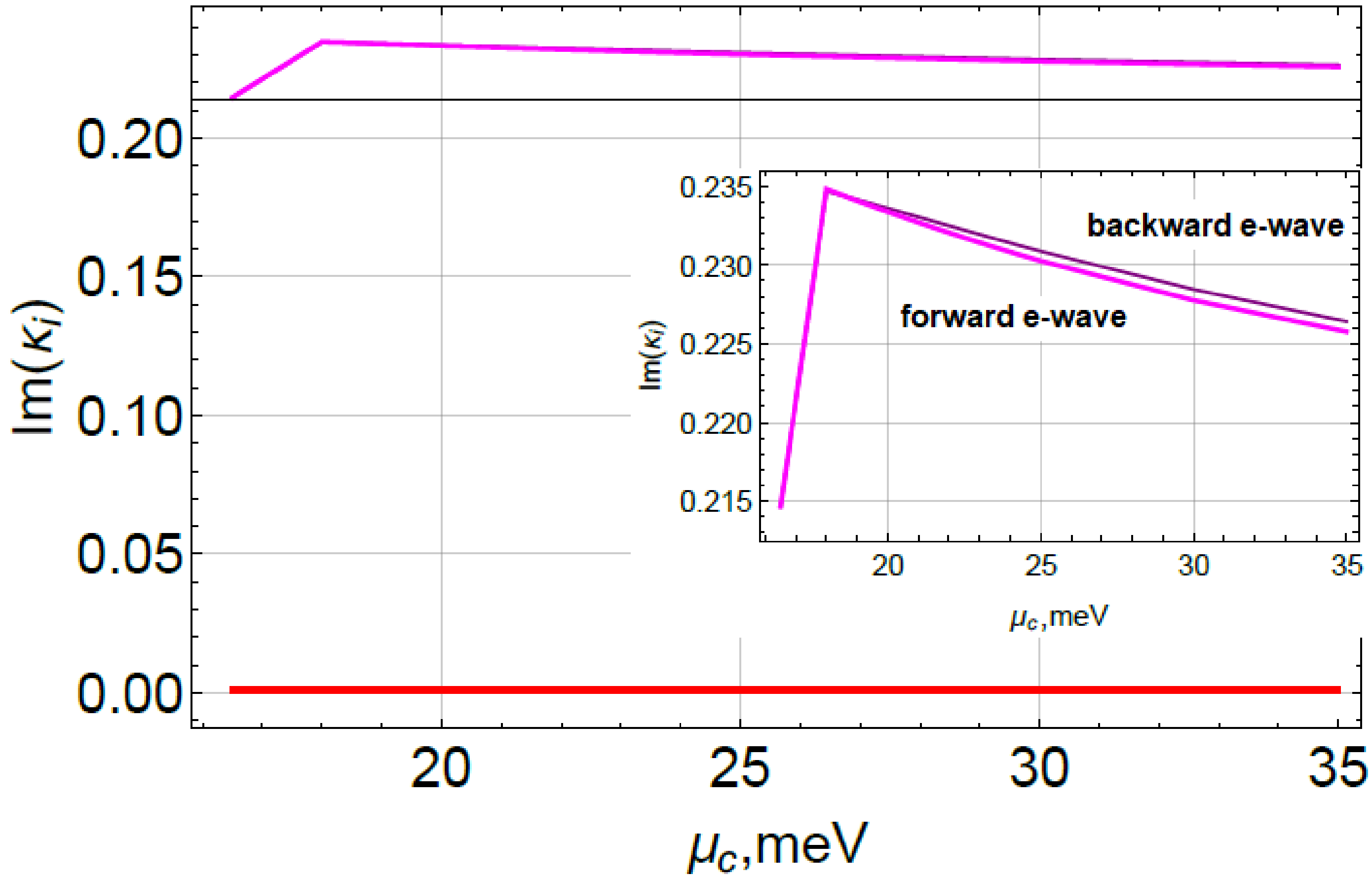
Black curves – $\mathrm{Re}(\varkappa_{i,k})$
Red curves – $\mathrm{Im}(\varkappa_{i,k})$

l_1=600μm, l_2=1320μm, h=5μm

$\varphi=\pi/2$, θ= 55°, α=15°

E_f = 25meV, $\tau = 10^{-12}$ s, T = 300° K

Im($\varkappa$) vs chemical potential (meV) of grapheme



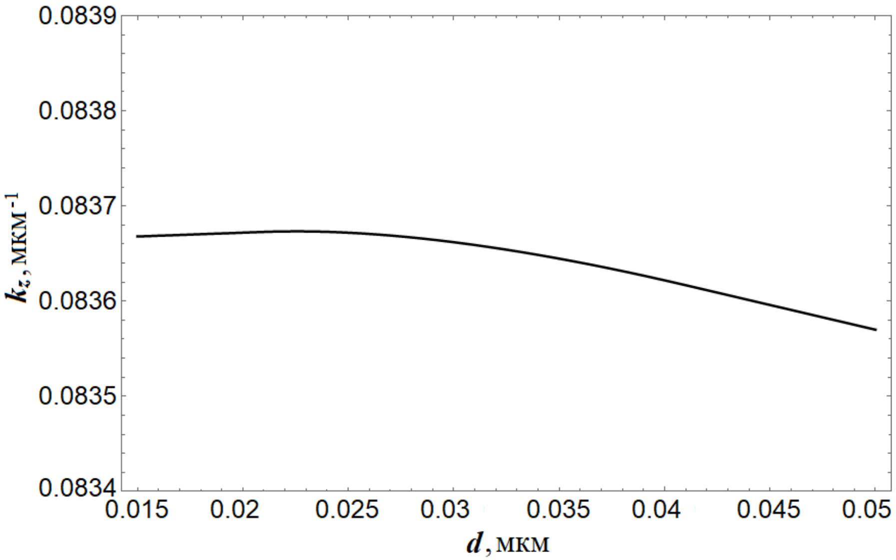
The net gain for the extraordinary waves Im($\varkappa_2$) and Im($\varkappa_3$) vs chemical potential (meV) of grapheme.
Level of losses (0.0009) given by red line.
k_z = 0.07445.

Dependence of radiation characteristics on the length of the outer part of the resonator *l*, characterized by loss.

The values of the *z*-component of the wave vector kz and the corresponding values of the generation frequency *f* for three values of the AGMM period *d*.

d, mkm	k_z	f, THz
0.05	0.083571	4.13093
0.03	0.083662	4.13548
0.0115	0.083668	4.13578

Dependences of Im(ϰi), which characterizes the amplification in the system, and kz on the length *l*.



Based on numerical simulation, it was determined that the generation line width is Δ*f* ≈ 0.00455 THz.

The generation frequency does not change significantly with changes in the value of the period. This fact is important both for estimating the width of the generation line and for the experimental implementation of the object under study due to the difficulties associated with the need to achieve ultra-small sizes of the proposed structures.

Conclusion

Some theoretical aspects of THz lasing in the cavity with an active graphene-based AHMM are presented:

- The gain in AHMM structure is provided by inversed population of carriers in graphene and takes place in the frequency range from 2.5 to 7 THz. . It was shown that that maximal net gain is achieved for an extraordinary mode under the value of chemical potential 25 meV.
- The frequency of oscillation and field intensity was calculated from the solution of equations for real and imaginary parts of log of eigenvalues of total transfer matrix of one period of the structure.
- Both forward and backward waves in the AHMM were included, this gives possibilities to investigate other than standing wave cavity configurations. We have demonstrated that four eigenmodes: two ordinary and two extraordinary, can be excited in a cavity filled with AHMM and all, 3, 2, or only one wave may contribute to laser oscillations
- The eigenwaves of the cavity at THz frequencies is calculated, accounting the saturation of the gain.
- It was assumed that gain saturation occurs due to the dependence of chemical potential of the graphene sheets upon the transverse component of the electric field of the THz radiation. We have shown that the gain saturation arises at the electric field strength about $2.7 \cdot 10^{12}$ V/m.
- This laser model allows to calculate the line width due to natural fluctuations of the field in the cavity and in ehe preliminary estimate is $\Delta f \approx 0.00455$ THz.