Terahertz laser based on a hyperbolic metamaterial consisting of thin layers of graphene

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Theoretical investigation of the terahertz lasing in the cavity containing graphene-based asymmetric hyperbolic metamaterial was present. The metamaterial under investigation is a nanoscale structure composed of periodically arranged layers of a semiconductor and inverted graphene. The aim of study is development of the component base for creating devices for terahertz generation and manipulation. The radiation characteristics in the cavity have been investigated by the transfer matrix method. A hyperbolic metamaterial is considered as a homogeneous medium with effective parameters due to the smallness of its period. The optimal conditions and parameters of the structure for an effective terahertz generation have been determined by numerical simulation. Generation linewidth, graphene saturation intensity, radiation power and the impact of the geometric parameters of structure on the process of terahertz wave generation have been estimated.
Cavity containing graphene-based asymmetric hyperbolic metamaterial.

Complex resonator with thin hyperbolic media inside.

Hyperbolic media is a nanoscale metamaterial composed of periodically arranged layers of a semiconductor (gray areas) and inverted graphene (red lines) with optics axis tilted with respect to outer boundary as an active media.
Hyperbolic media

Hyperbolic medium exhibits hyperbolic-type dispersion in space of wave-vectors and has the diagonal extremely anisotropic permittivity tensor. The dispersive properties of the hyperbolic metamaterials are inherent to uniaxial materials whose axial and tangential permittivity components are of different signs.

\[
\begin{bmatrix}
\varepsilon_{xx} & 0 & 0 \\
0 & \varepsilon_{yy} & 0 \\
0 & 0 & \varepsilon_{zz}
\end{bmatrix}
\]


Magnetized plasma (for RF)

Natural objects

Graphite (for UV)

Metamaterials

\[
\begin{align*}
\varepsilon_{xx} &= \varepsilon_{yy} > 0 \\
\varepsilon_{zz} &= < 0 \\
\varepsilon_{xx} &= \varepsilon_{yy} < 0 \\
\varepsilon_{zz} &= > 0
\end{align*}
\]

Asymmetric Hyperbolic Metamaterials – parameters

\[ \mathbf{\varepsilon} = \begin{bmatrix} \varepsilon_{\parallel} & 0 & 0 \\ 0 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\perp} \end{bmatrix} \]

\[ \varepsilon_{\perp} = \varepsilon_{\parallel} + i \frac{\sigma(\omega)}{d \omega \varepsilon_0} \]

\[ \varepsilon_{\parallel} = \varepsilon_h = \varepsilon_{SiC} \]

Kubo model of graphen conductivity

\[
\sigma_e(\omega, E_0) = \frac{-ie^2 k_B T}{\pi \hbar^2 (\omega - i2\Gamma)} \left[ \frac{\mu_e(E_0)}{k_B T} + 2 \ln \left( \frac{\exp\left(\frac{-\mu_e(E_0)}{k_B T}\right) + 1}{\exp\left(\frac{-\mu_e(E_0)}{k_B T}\right) - 1}\right) \right] \\
= \frac{ie^2 (\omega - i2\Gamma)^2}{\pi \hbar^2} \left[ \exp\left(\frac{-\mu_e(E_0)}{k_B T} + 1\right) \right]^{-1} \left[ \exp\left(\frac{-\mu_e(E_0)}{k_B T} - 1\right) \right]^{-1} d\xi
\]

Permittivity of SiC

\[
\varepsilon_k(\omega) = \varepsilon_{ke} + \frac{\omega_{kp}^2}{\omega_{kT}^2 - \omega^2 - i\gamma_k \omega}
\]

\[
\omega_{kp}^2 = \varepsilon_{ke} (\omega_{kLO}^2 - \omega_{kTD}^2)
\]

\[
\varepsilon_{SiC} = \frac{\varepsilon_{k=p} + \varepsilon_{k=t}}{2}
\]


Frequency region from 2.5 to 7 THz is suitable for THz amplification

\[ E_F = 25 \text{ meV}, \tau = 10^{-12} \text{ c}, \; T = 300^\circ \text{ K} \]

\[ \text{Re}(\sigma_{gr}) = -0.16 \text{ mCm}, \; f = 5.8 \text{ THz} \]

\[ \text{Im}(\varepsilon_{\perp}) \] characterizes the gain properties of HMM:

Re[\( \sigma_{gr} (\omega) \)] < 0 energy gain
Re[\( \sigma_{gr} (\omega) \)] > 0 energy loss


- О. Н. Козина, Л. А. Мельников. Графеновая гиперболическая наноструктура для генерации терагерцевой волны. Радиотехника и электроника, 2022, том 67, № 10, с. 1–5.
Asymmetric Hyperbolic Metamaterials – parameters with taking into account the saturation of amplification in graphene

The dependence of chemical potential (meV) of graphene from transverse electric field (V/nm). $E_0$ is a component of the external electric strength vector transverse to the graphene plane

$$E_0 = \frac{e}{\pi \hbar^2 v_F^2 \varepsilon_b} \int_0^\infty d\varepsilon \left( f_d(\varepsilon) - f_d(\varepsilon + 2\mu_c) \right)$$

$f_d(\varepsilon) = 1 / (\exp[\varepsilon - \mu_c / k_B T] + 1)$

$\sigma = \sigma_c(\mu_c(E_0))$

$\varepsilon_\perp = \varepsilon_\parallel + \frac{i}{d\omega \varepsilon_0} \left[ \frac{\sigma'(\omega, E_0)}{1 + S} + i\sigma''(\omega, E_0) \right]$
Cavity partially filled with AHMM

Total round-trip transfer matrix

\[ P_t = P_0(l)P(h) \]

\[ \Psi_T = \mathbf{P}(h)(\Psi_I + \Psi_R) \]

\[ \Delta_{i,j} = f(\epsilon_\perp, \epsilon_\parallel, \theta, \phi, \psi, k_x), \quad i, j = 1, 2, 3, 4 \]

Due to the linearity of the eigenvalues problem at a given \( E_0 \) the eigenvalues of the entire transfer matrix \( P_t \) are

\[ \Lambda_i = e^{(i \xi_i L)}, \]

where \( \xi_i = \ln \Lambda_i \) characterizes the phase delay at one pass \( (L = l + h) \).

Mode frequency and mode intensity proportional to \( E_0 \) are the solutions of the equations:

\[ \text{Re}[\kappa_i(k_z, E)] = 0, \quad \text{Im}[\kappa_i(k_z, E)] = 0 \]

These equations may be solved numerically to find \( k_{z0} \) and \( E_0 \).

Due to a weak dependence of \( k_{z0} \) from \( E_0 \), \( k_{z0} \) can be easily derived from the first equation, and \( E_0 \) from the second equation.
Eigenwaves in the cavity partially filled with AHMM

\[ \Lambda_i = e^{(i \xi_i L)} \cdot \text{eigenvalues of the entire transfer matrix } P_t \text{ at a given } E_0 \]

\[ \xi_i = \ln \Lambda_i \text{ characterizes the phase delay at one pass } (L = l + h) \]

\[ \text{Re}(\xi) = 2\pi m, \quad m = 0, \pm 1, \pm 2, \ldots \text{ determines the eigen frequencies} \]

\[ \text{Re}[\kappa_i(k_z, E)] = 0, \quad \text{Im}[\kappa_i(k_z, E)] = 0 \]

0.05 < k_z < 0.14 corresponds to 2.5 до 7 ТГц

\[ l_1 = 600 \, \mu\text{m}, \quad l_2 = 1320 \, \mu\text{m} \]

\[ h = 5 \, \mu\text{m} \]

\[ d = 50 \, \text{nm} \]
Eigenvalues $\bm{\kappa_{i,k}}$ vs $k_z$

Mode generation condition determined by the equation

$$\text{Re}[\bm{\kappa_{i,k}(k_z)}] = 0$$

Black curves – Re($\bm{\kappa_{i,k}}$)
Red curves – Im($\bm{\kappa_{i,k}}$)

$l_1=600\mu m, l_2=1320\mu m, h=5\mu m$

$\varphi=\pi/2, \theta=55^\circ, \alpha=15^\circ$

$E_f=25\text{meV}, \tau=10^{-12}\text{s}, T=300^\circ\text{K}$
The net gain for the extraordinary waves $\text{Im}(\kappa_2)$ and $\text{Im}(\kappa_3)$ vs chemical potential (meV) of grapheme. Level of losses (0.0009) given by red line. $k_z = 0.07445$. 

$\text{Im}(\kappa)$ vs chemical potential (meV) of grapheme.
Dependence of radiation characteristics on the length of the outer part of the resonator $l$, characterized by loss.

The values of the $z$-component of the wave vector $k_z$ and the corresponding values of the generation frequency $f$ for three values of the AGMM period $d$.

<table>
<thead>
<tr>
<th>$d$, mkm</th>
<th>$k_z$</th>
<th>$f$, THz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.083571</td>
<td>4.13093</td>
</tr>
<tr>
<td>0.03</td>
<td>0.083662</td>
<td>4.13548</td>
</tr>
<tr>
<td>0.0115</td>
<td>0.083668</td>
<td>4.13578</td>
</tr>
</tbody>
</table>

Dependences of $\text{Im}(\chi_i)$, which characterizes the amplification in the system, and $k_z$ on the length $l$.

Based on numerical simulation, it was determined that the generation line width is $\Delta f \approx 0.00455$ THz.

The generation frequency does not change significantly with changes in the value of the period. This fact is important both for estimating the width of the generation line and for the experimental implementation of the object under study due to the difficulties associated with the need to achieve ultra-small sizes of the proposed structures.
Conclusion

Some theoretical aspects of THz lasing in the cavity with an active graphene-based AHMM are presented:

• The gain in AHMM structure is provided by inversed population of carriers in graphene and takes place in the frequency range from 2.5 to 7 THz. It was shown that that maximal net gain is achieved for an extraordinary mode under the value of chemical potential 25 meV.

• The frequency of oscillation and field intensity was calculated from the solution of equations for real and imaginary parts of log of eigenvalues of total transfer matrix of one period of the structure.

• Both forward and backward waves in the AHMM were included, this gives possibilities to investigate other than standing wave cavity configurations. We have demonstrated that four eigenmodes: two ordinary and two extraordinary, can be excited in a cavity filled with AHMM and all, 3, 2, or only one wave may contribute to laser oscillations

• The eigenwaves of the cavity at THz frequencies is calculated, accounting the saturation of the gain.

• It was assumed that gain saturation occurs due to the dependence of chemical potential of the graphene sheets upon the transverse component of the electric field of the THz radiation. We have shown that the gain saturation arises at the electric field strength about $2.7 \cdot 10^{12}$ V/m.

• This laser model allows to calculate the line width due to natural fluctuations of the field in the cavity and in ehe preliminary estimate is $\Delta f \approx 0.00455$ THz.