Sudden death of entanglement in the three-qubit many-photon cavity quantum electrodynamics model

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1. The model and its exact solution

We consider three identical two-level natural or artificial atoms (qubits) A, B and C. The atoms B and C are trapped in a two single-mode infinite-Q cavities and resonantly interacting with cavities fields through the m-photon transitions. The atom A is outside the cavities and there is no interaction between the cavities fields and atom A.

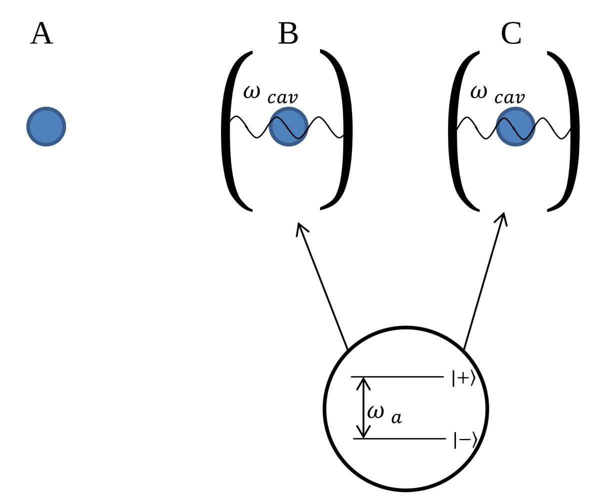

Figure 1: *The schematic diagram of the model used in this paper for $m = 1$. Here ω_{cav} is the cavity mode frequency, ω_a is frequency of transition between levels. The ground and excited states of atoms are denoted by $|-\rangle$ and $|+\rangle$ respectively.*

The interaction Hamiltonian of the system under consideration in the standard approximations has the following form

$$\hat{H}_I = \hbar\gamma\left(\hat{\sigma}_B^+\hat{b}^m + \hat{\sigma}_B^-\hat{b}^{+m}\right) + \hbar\gamma\left(\hat{\sigma}_C^+\hat{c}^m + \hat{\sigma}_C^-\hat{c}^{+m}\right), \qquad (1)$$

where $\hat{\sigma}_i^+ = |+\rangle_{ii}\langle-|$ and $\hat{\sigma}_i^- = |-\rangle_{ii}\langle+|$ are the rasing and the lowering operators in the i-th qubit $(i = B, C)$, $\hat{b}^+(\hat{c}^+)$ and $\hat{b}(\hat{c})$ are the creation and annihilation operators of the cavity photons $n_B(n_C)$, γ is the qubit-field coupling, m is the photon multiple of transitions.

As the initial state of the resonators field, we choose a thermal state with a density matrix of the form:

$$\Xi_{F_{n_B}}(0) = \sum_{n_B} p_{n_B}|n_B\rangle\langle n_B|, \Xi_{F_{n_C}}(0) = \sum_{n_C} p_{n_C}|n_C\rangle\langle n_C|. \qquad (2)$$

There are weight coefficients

$$p_{n_B} = \frac{\bar{n}_B^{n_B}}{(\bar{n}_B + 1)^{n_B+1}}, \; p_{n_C} = \frac{\bar{n}_C^{n_C}}{(\bar{n}_C + 1)^{n_C+1}}; \bar{n}_{B(C)} = (exp[\hbar\omega_{cav}/kT_{B(C)}] - 1)^{-1}$$

the average number of photons of the resonator, $T_{B(C)}$ is the cavities temperature.

Let the initial states of qubits be the W-type genuine entangled states such as

$$|W_1(0)\rangle_{ABC} = x_2|+,+,-\rangle + y_2|+,-,+\rangle + z_2|-,+,+\rangle, \qquad (3)$$

$$|W_2(0)\rangle_{ABC} = x_1|-,-,+\rangle + y_1|-,+,-\rangle + z_1|+,-,-\rangle, \qquad (4)$$

with $|x_1|^2 + |y_1|^2 + |z_1|^2 = 1$, $|x_2|^2 + |y_2|^2 + |z_2|^2 = 1$.

We derived the solutions of the quantum Liouville equation for the initial states of qubits (3)-(4) and the thermal field of resonators (2) in the model (1):

$$i\hbar\frac{\partial \Xi_{ABCF_{n_B}F_{n_C}}}{\partial t} = \left[\hat{H}_I, \Xi_{ABCF_{n_B}F_{n_C}}\right]. \qquad (5)$$

Here $\Xi_{ABCF_{n_B}F_{n_C}}$ is a density matrix including three qubits and two resonator field modes.

2. Calculation of the entanglement criterion

To calculate the various known criteria for the entanglement of three-qubit systems, we will need to calculate the reduced density matrices of a system of two and three qubits. To obtain a three - qubit density matrix $\Xi_{ABC}(t)$, it is enough to calculate the trace of the density matrix of the entire system (5) from the variable fields of the resonator

$$\Xi_{ABC}(t) = Tr_{F_{n_B}}Tr_{F_{n_C}}\Xi_{ABCF_{n_B}F_{n_C}}. \qquad (6)$$

To calculate the two-qubit density matrix, it will be necessary to average the three-qubit density matrix (6) over the variables of the third qubit, i.e.

$$\Xi_{ij}(t) = Tr_k\Xi_{ABC}(t)(i, j, k = A, B, C; i \neq j, j \neq k, i \neq k). \qquad (7)$$

When studying the entanglement of qubits in the considered model for genuine entangled W-type states, we will use the criterion of negativity of qubit pairs as a quantitative criterion of entanglement. We define negativity for qubits i and j in a standard way:

$$\varepsilon_{ij} = -2\sum_k (\lambda_{ij})_k^-, \qquad (8)$$

where λ_{ij} are the negative eigenvalues of a reduced two-qubit density matrix (7) partially transposed over variables of one qubit $\Xi_{ij}^T(t)$, which has the following form for states (3)-(4):

$$\Xi_{ij}^T(t) = \begin{pmatrix} \Xi_{11}^{ij} & 0 & 0 & \Xi_{32}^{ij} \\ 0 & \Xi_{22}^{ij} & 0 & 0 \\ 0 & 0 & \Xi_{33}^{ij} & 0 \\ \Xi_{23}^{ij} & 0 & 0 & \Xi_{44}^{ij} \end{pmatrix}, \begin{pmatrix} |+_i, +_j\rangle \\ |+_i, -_j\rangle \\ |-_i, +_j\rangle \\ |-_i, -_j\rangle \end{pmatrix} \longleftrightarrow \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}. \qquad (9)$$

Then, given expression (9), the formula for the negativity criterion will be written as:

$$\varepsilon_{ij} = \sqrt{(\Xi_{44}^{ij} - \Xi_{11}^{ij})^2 + 4|\Xi_{23}^{ij}|^2} - \Xi_{11}^{ij} - \Xi_{44}^{ij}. \qquad (10)$$

3. Computer modeling and results

The results of computer modeling of the pairwise negativities for genuine entangled qubits states (3)–(4) and thermal field (2) are shown in Figs. 2-3.

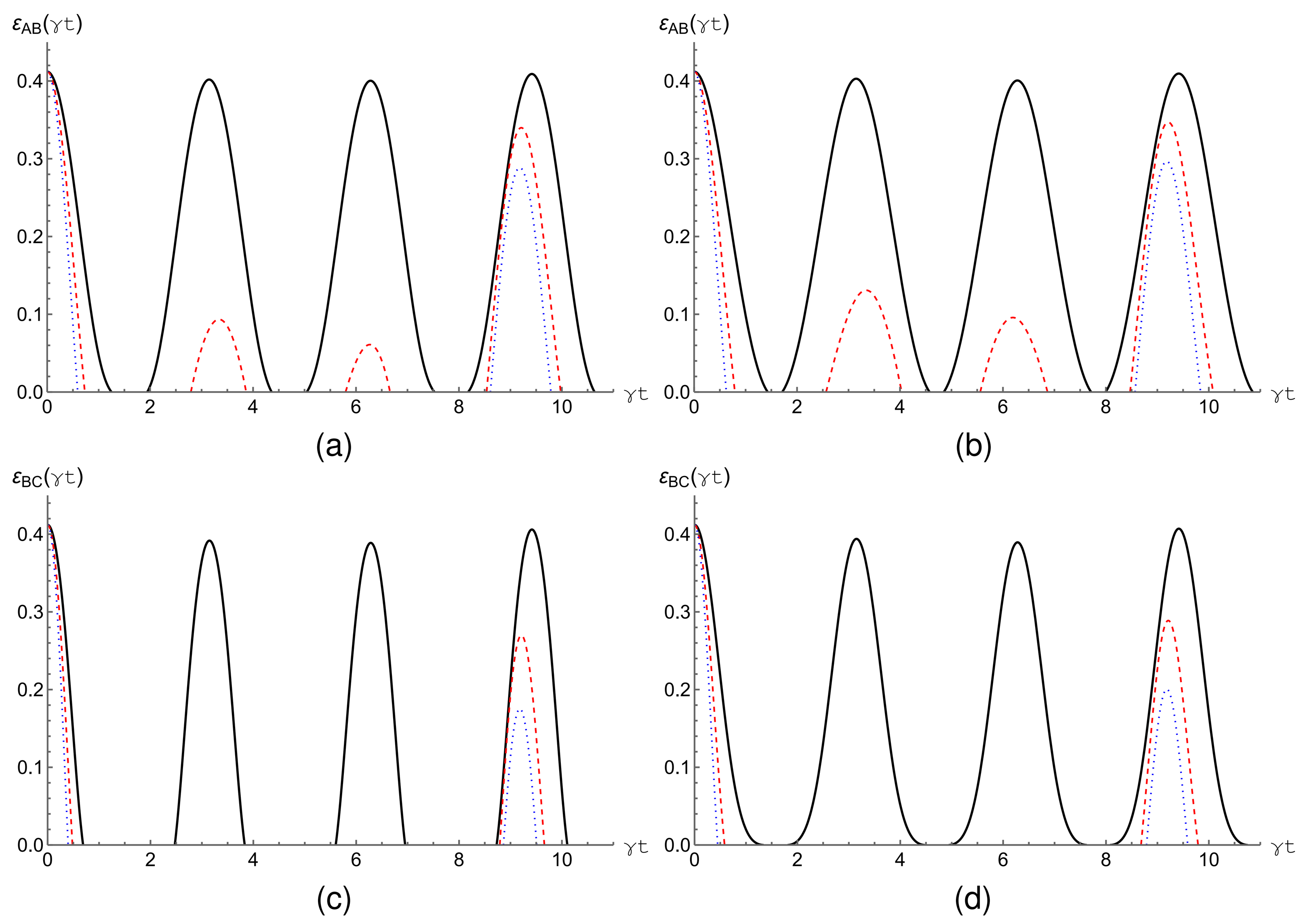

Figure 2: *The negativity $\varepsilon_{AB(AC)}(\gamma t)$ (a,b) and $\varepsilon_{BC}(\gamma t)$ (c,d) are plotted as a functions of scaled time γt for the initial states (3)–(4) with $x_{1,2} = y_{1,2} = z_{1,2} = 1/\sqrt{3}$. The mean number of thermal photons: $\bar{n}_B = \bar{n}_C = 0.01$ (black solid line), $\bar{n}_B = \bar{n}_C = 0.5$ (red dashed line), $\bar{n}_B = \bar{n}_C = 1$ (blue dotted line). The photon multiple $m = 1$.*

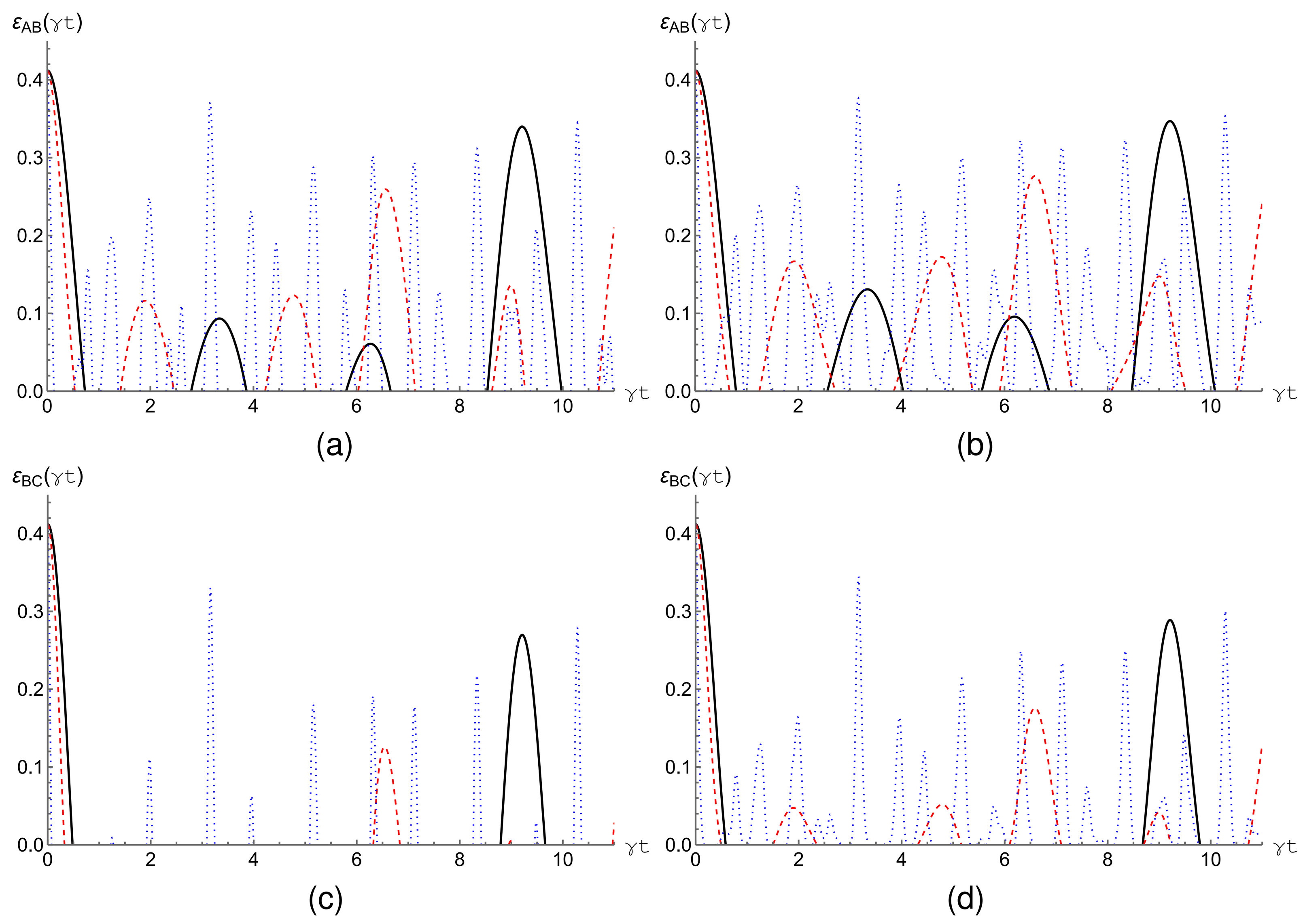

Figure 3: *The negativity $\varepsilon_{AB(AC)}(\gamma t)$ (a,b) and $\varepsilon_{BC}(\gamma t)$ (c,d) are plotted as a functions of scaled time γt for the initial states (3)–(4) with $x_{1,2} = y_{1,2} = z_{1,2} = 1/\sqrt{3}$. The mean number of thermal photons $\bar{n}_B = \bar{n}_C = 0.5$. The photon multiple m: $m = 1$ (black solid line), $m = 2$ (red dashed line), $m = 4$ (blue dotted line).*

- An analysis of computer modeling of the pairwise negativities for genuine entangled qubits states (3)–(4) and thermal field (2) are shown in figures 2-3 that with increasing thermal noise intensity, the maximum amount of entanglement of both the pairs of qubits decreases for any parameters model.
- For m-photon transitions the negativity vanishes at some discrete time moments. This implies that there is ESD for the atoms A and B (or A and C) and B and C. The time of ESD decreases with photon multiple growth, i.e. this can be controlled by the parameter m. This is true for both genuine entangled states except for atoms B and C of the initial state $|W_1(0)\rangle_{ABC}$.
- The main difference between the two genuine entangled states $|W_1(0)\rangle_{ABC}$ (a,c) and $|W_2(0)\rangle_{ABC}$ (b,d) is as follows. For a model with one-photon and many-photon transitions, the maximum degree of entanglement is greater for a genuine entangled state of the form $|W_2(0)\rangle_{ABC}$. Moreover, a comparison of the graphs shows that the time intervals during which the effect of sudden death of entanglement occurs are significantly longer for the state $|W_1(0)\rangle_{ABC}$. Thus, we conclude that the initial genuinely entangled state is $|W_2(0)\rangle_{ABC}$ is more resistant to the destructive effect of the thermal field.