Entanglement in three-qubit nonlinear Tavis-Cummings model

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1. The model and its exact solution

We consider three identical two-level natural or artificial atoms (qubits) Q_1, Q_2 and Q_3, which are trapped in a one-mode resonator with infinite Q-factor and resonantly interact with the resonator field through m-photon transitions. The configuration is shown in Fig. 1.

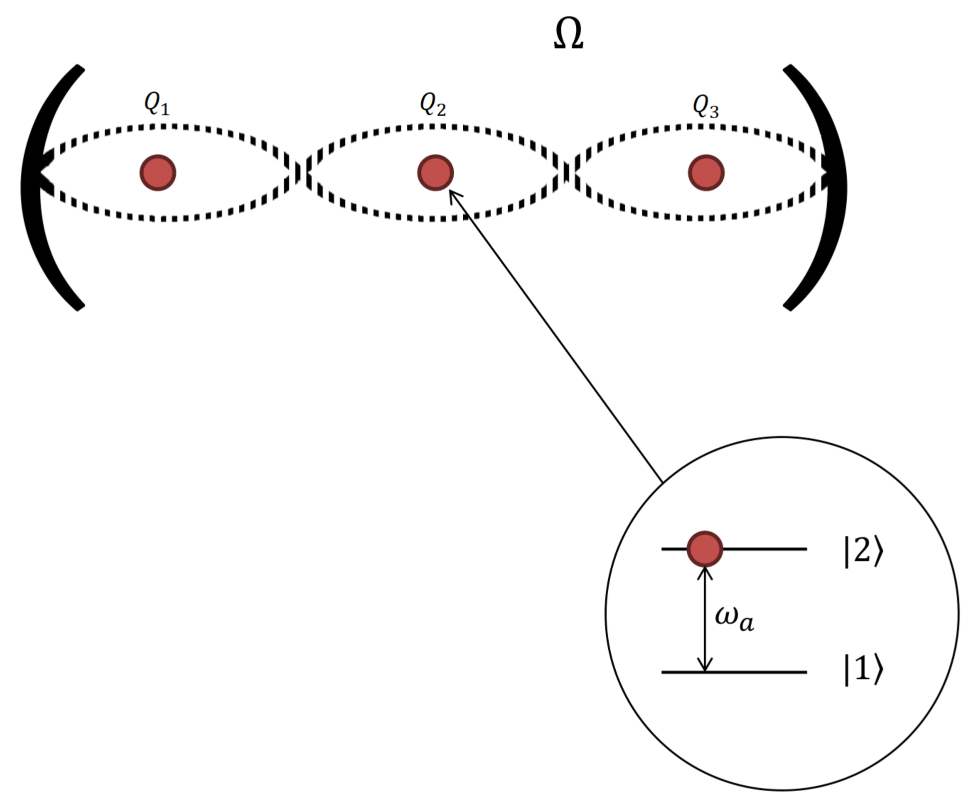


Figure 1: *The schematic diagram of the model used in this paper for $m = 1$. Here Ω is the cavity mode frequency, ω_a is frequency of transition between levels. The ground and excited states of atoms are denoted by $|1\rangle$ and $|2\rangle$ respectively.*

The interaction Hamiltonian of the system under consideration in the standard approximations has the following form

$$\hat{H}_I = \hbar\gamma \sum_i \left(\hat{\sigma}_i^- \hat{a}^{+m} + \hat{\sigma}_i^+ \hat{a}^m\right), \tag{1}$$

where $\hat{\sigma}_i^+ = |2\rangle_{ii}\langle 1|$ and $\hat{\sigma}_i^- = |1\rangle_{ii}\langle 2|$ are the rasing and the lowering operators in the i-th qubit ($i = 1, 2, 3$), $\hat{a}^+$ and $\hat{a}$ are the creation and annihilation operators, γ is the qubit-field coupling, m is the photon multiple of transitions.
As the initial state of the resonator field, we choose a thermal state with a density matrix of the form:

$$\Xi_{F_n}(0) = \sum_n p_n |n\rangle\langle n|. \tag{2}$$

There are weight coefficients

$$p_n = \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}; \bar{n} = (exp[\hbar\Omega/kT] - 1)^{-1}$$

the average number of photons of the resonator, T is the cavitiy temperature.
As the initial states of the qubits, we choose the following

$$|\psi_1(0)\rangle_{Q_1Q_2Q_3} = |2, 2, 1\rangle, \tag{3}$$

$$|\psi_2(0)\rangle_{Q_1Q_2Q_3} = \cos\alpha|1, 2, 1\rangle + \sin\alpha|1, 1, 2\rangle, \tag{4}$$

$$|W_1(0)\rangle_{Q_1Q_2Q_3} = \cos\theta|2, 2, 1\rangle + \sin\theta\sin\varphi|2, 1, 2\rangle + \sin\theta\cos\varphi|1, 2, 2\rangle, \tag{5}$$

$$|W_2(0)\rangle_{Q_1Q_2Q_3} = \cos\theta|1, 1, 2\rangle + \sin\theta\sin\varphi|1, 2, 1\rangle + \sin\theta\cos\varphi|2, 1, 1\rangle, \tag{6}$$

where α, θ and φ is parameters that determine the initial degree of entanglement.
We derived the solutions of the quantum Liouville equation for the initial states of qubits (3)-(6) and the thermal field of resonators (2) in the model (1):

$$i\hbar\frac{\partial \Xi_{Q_1Q_2Q_3F_n}}{\partial t} = \left[\hat{H}_I, \Xi_{Q_1Q_2Q_3F_n}\right]. \tag{7}$$

Here $\Xi_{Q_1Q_2Q_3F_n}$ is a density matrix including three qubits and resonator field mode.

2. Calculation of the entanglement criterion

To calculate the various known criteria for the entanglement of three-qubit systems, we will need to calculate the reduced density matrices of a system of two and three qubits. To obtain a three - qubit density matrix $\Xi_{Q_1Q_2Q_3}(t)$, it is enough to calculate the trace of the density matrix of the entire system (7) from the variable fields of the resonator

$$\Xi_{Q_1Q_2Q_3}(t) = Tr_{F_n}\Xi_{Q_1Q_2Q_3F_n}. \tag{8}$$

To calculate the two-qubit density matrix, it will be necessary to average the three-qubit density matrix (8) over the variables of the third qubit, i.e.

$$\Xi_{Q_iQ_j}(t) = Tr_{Q_k}\Xi_{Q_1Q_2Q_3}(t)(i, j, k = 1, 2, 3; i \neq j, j \neq k, i \neq k). \tag{9}$$

When studying the entanglement of qubits in the considered model for initial states (3)-(6), we will use the criterion of negativity of qubit pairs as a quantitative criterion of entanglement. We define negativity for qubits Q_i and Q_j in a standard way:

$$\varepsilon_{ij} = -2\sum_k (\lambda_{ij})_k^-, \tag{10}$$

where λ_{ij} are the negative eigenvalues of a reduced two-qubit density matrix (9) partially transposed over variables of one qubit $\Xi^T_{Q_iQ_j}(t)$, which has the following form for states (3)-(6):

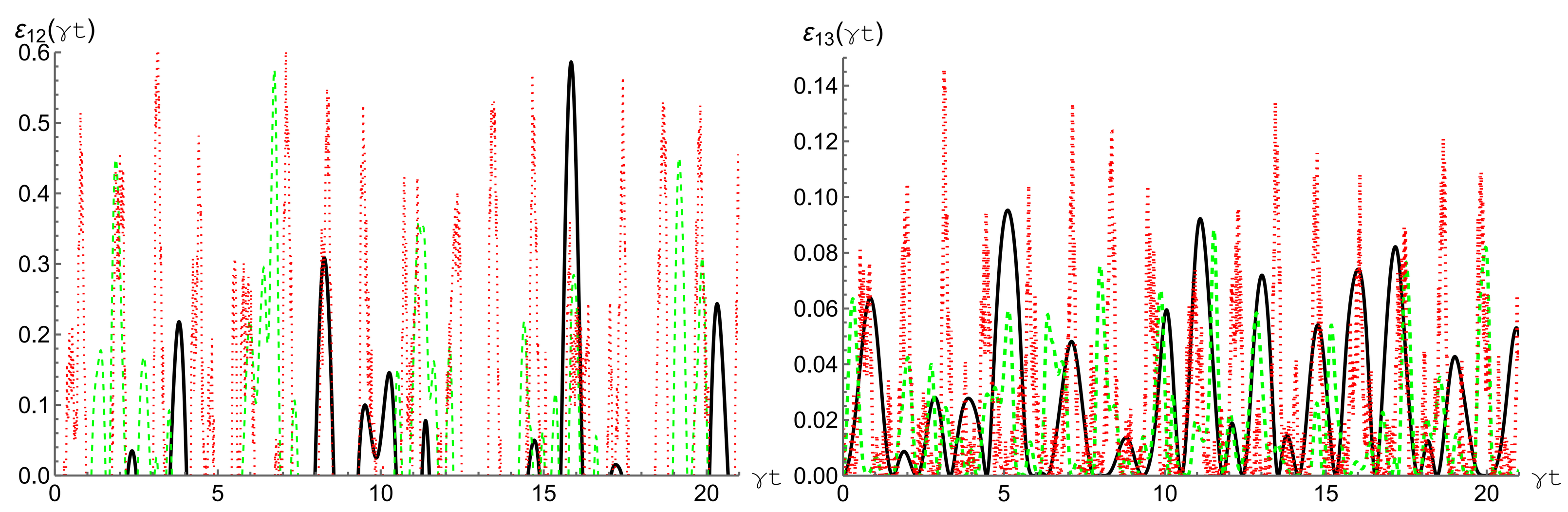
$$\Xi^T_{Q_iQ_j}(t) = \begin{pmatrix} \Xi^{ij}_{11} & 0 & 0 & \Xi^{ij}_{32} \\ 0 & \Xi^{ij}_{22} & 0 & 0 \\ 0 & 0 & \Xi^{ij}_{33} & 0 \\ \Xi^{ij}_{23} & 0 & 0 & \Xi^{ij}_{44} \end{pmatrix}, \begin{pmatrix} |+_i, +_j\rangle \\ |+_i, -_j\rangle \\ |-_i, +_j\rangle \\ |-_i, -_j\rangle \end{pmatrix} \longleftrightarrow \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}. \tag{11}$$

Then, given expression (11), the formula for the negativity criterion (10) will be written as:

$$\varepsilon_{ij} = \sqrt{(\Xi^{ij}_{44} - \Xi^{ij}_{11})^2 + 4|\Xi^{ij}_{23}|^2} - \Xi^{ij}_{11} - \Xi^{ij}_{44}. \tag{12}$$

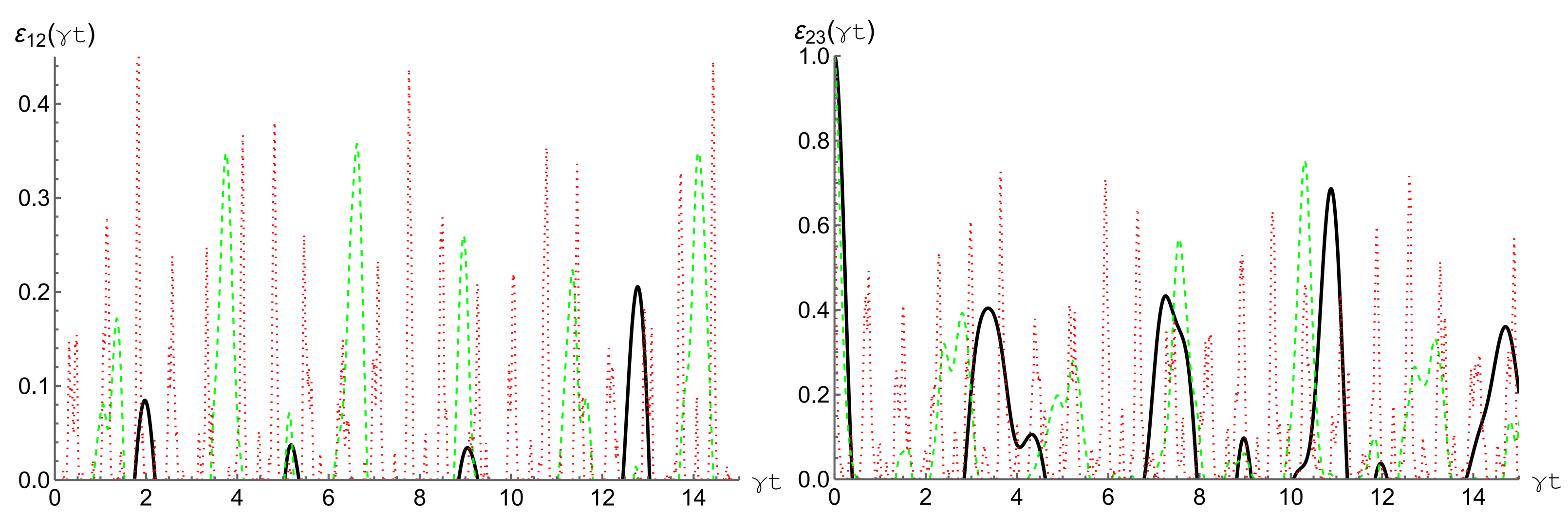
3. Computer modeling and results

The results of computer modeling of the pairwise negativities for initial qubits states (3)–(6) and thermal field (2) are shown in Figs. 2-4.



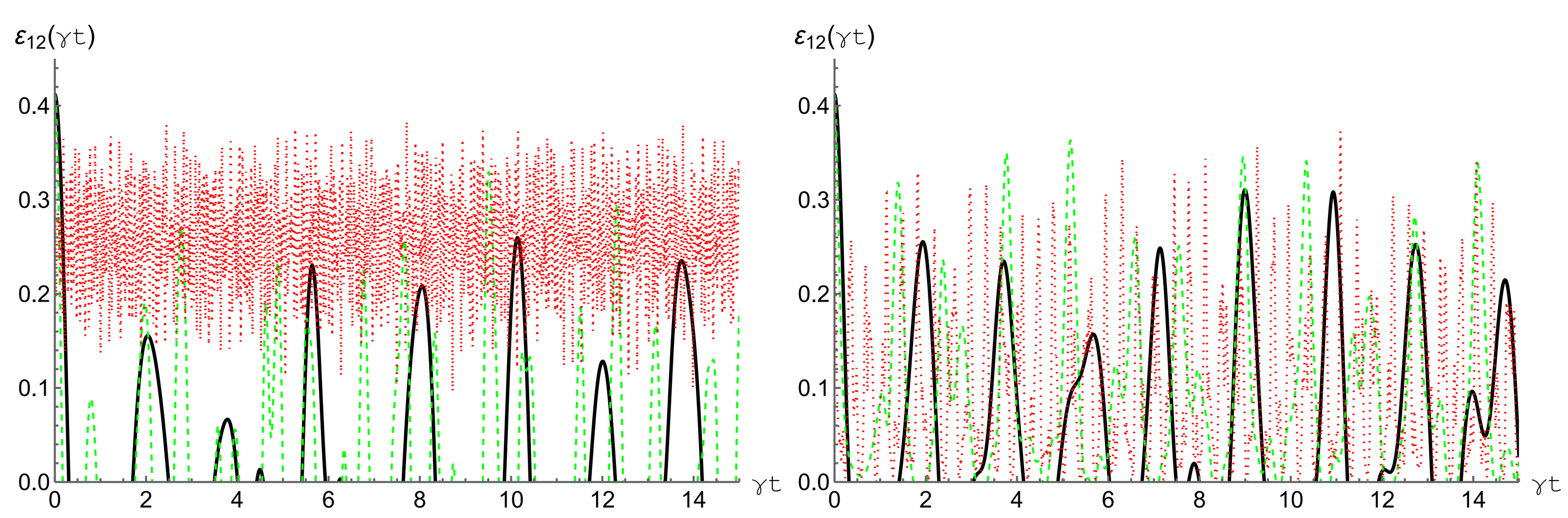
(a) (b)

Figure 2: *The negativity $\varepsilon_{12}(\gamma t)$ (a) and $\varepsilon_{13}(\gamma t)$ (b) are plotted as a functions of scaled time γt for the initial state (3). The mean number of thermal photons: $\bar{n} = 1$. The photon multiple: $m = 1$ (black solid line), $m = 2$ (green dashed line), $m = 4$ (red dotted line).*



(a) (b)

Figure 3: *The negativity $\varepsilon_{12}(\gamma t)$ (a) and $\varepsilon_{23}(\gamma t)$ (b) are plotted as a functions of scaled time γt for the initial states (4) with $\alpha = \pi/4$. The mean number of thermal photons $\bar{n} = 1$. The photon multiple m: $m = 1$ (black solid line), $m = 2$ (green dashed line), $m = 4$ (red dotted line).*



(a) (b)

Figure 4: *The negativity $\varepsilon_{12}(\gamma t)$ are plotted as a functions of scaled time γt for the initial state $|W_1(0)\rangle_{ABC}$ (a) and for the initial state $|W_2(0)\rangle_{ABC}$ (b) with $\theta = Arccos[1/\sqrt{3}]$, $\varphi = \pi/4$. The mean number of thermal photons $\bar{n} = 1$. The photon multiple m: $m = 1$ (black solid line), $m = 2$ (green dashed line), $m = 4$ (red dotted line).*

- The analysis of computer modeling of the negativity criterion for the initial states of qubits (3)–(6) and the thermal field (2) obtained as a result of computer modeling shows in Figures 2-4 that with an increase in the multiples parameter of photon transitions m, the maximum degree of entanglement.
- For m-photon transitions, the negativity vanishes at some discrete time moments. This means that there is an sudden death of entanglement. The time of sudden death of entanglement decreases with an increase in the photon transition multiples parameter, i.e. this can be controlled using the m parameter. Moreover, for many-photon processes with large m the phenomenon of sudden death of entanglement can be eliminated (see Figure 4 (a)). Thus, for large values of photon transition multiples the oscillation of negativity decreases and we obtain the long-lived genuine entangled W-states even for sufficiently intense fields.