QUANTUM OPTICS AND DYNAMICAL GROUPS THEORY

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Abstract

The history of applications of the methods of dynamical Lie groups to the quantum theory of the interaction of radiation with matter is considered..

Dynamical symmetry

$$\hat{H} = \sum_{s_1,\dots,s_r} \omega_{s_1\dots s_r} \hat{A}_1^{s_1} \cdots \hat{A}_r^{s_r}$$

$$[\widehat{A}_k, \widehat{A}_l] = i\, C_{kl}^m \widehat{A}_m$$

Holomorphic functions representation

$$|\Psi> \longmapsto \Psi(z) = < z|\Psi > / < z|0 >, \quad |z> \equiv |\xi(z^1,\dots,z^n)>,$$

$$\Psi(z) = \int_{\mathcal{X}} K(z,\bar{w})\, \Psi(w)\, \exp[-\rho(w,\bar{w})]\, d\mu(w,\bar{w}),$$

$$K(z,\bar{w}) = < z|w > / < z|0 >< 0|w >$$

$$\omega^2 = i \sum_{\alpha,\beta} \frac{\partial^2 \ln K(z,\bar{z})}{\partial z^\alpha \partial \bar{z}^\beta} dz^\alpha \wedge d\bar{z}^\beta$$

$$\left(\hat{F}\,\Psi\right)(z) = \int_{\mathcal{X}} \mathcal{F}(z,\bar{w}) \frac{K(z,\bar{w})}{K(w,\bar{w})} \Psi(w) d\mu(w,\bar{w}),$$

G-invariant Kähler's 2-form

$$\mathcal{F}(z,\bar{w}) = \frac{< z|F|w >}{< z|w >}.$$

$$\hat{F} = \hat{F}_1 \cdot \hat{F}_2, \longrightarrow \mathcal{F}(z,\bar{z}) = \int_{\mathcal{X}} \mathcal{F}_1(z,\bar{w})\, \mathcal{F}_2(w,\bar{z}) \frac{K(z,\bar{w})\, K(w,\bar{z})}{K(z,\bar{z})\, K(w,\bar{w})} d\mu(w,\bar{w}).$$

$$\{\mathcal{F}_1, \mathcal{F}_2\}(z,\bar{z}) = \frac{i}{\hbar} \sum_{\alpha,\bar{\beta}} g^{\alpha\bar{\beta}} \left[\frac{\partial \mathcal{F}_1}{\partial z^\alpha} \frac{\partial \mathcal{F}_2}{\partial \bar{z}^\beta} - \frac{\partial \mathcal{F}_1}{\partial \bar{z}^\beta} \frac{\partial \mathcal{F}_2}{\partial z^\alpha}\right]$$

Open systems

Coherent relaxation of N-level systems, (the Markovian approximation)

$$\hat{H} = \hat{H}_A + \hat{H}_B + \hat{H}_{int} \qquad \hat{\mathcal{R}}(t) = \hat{\rho}_A(t) \otimes \hat{\rho}_B(0) \qquad \hat{\rho}(t) = tr_B[\hat{\mathcal{R}}(t)]$$

$$\begin{aligned}\frac{\partial \hat{\rho}}{\partial t} = \sum_{a=1}^{M} \frac{1}{2}\gamma_a \big[& (\mathcal{N}_a + 1)\,(2\,\hat{E}_a^- \,\hat{\rho}\, \hat{E}_a^+ - \hat{E}_a^+ \hat{E}_a^- \hat{\rho} - \hat{\rho}\, \hat{E}_a^+ \hat{E}_a^-) + \\ & + \mathcal{N}_a\,(2\,\hat{E}_a^+ \,\hat{\rho}\, \hat{E}_a^- - \hat{E}_a^- \hat{E}_a^+ \hat{\rho} - \hat{\rho}\, \hat{E}_a^- \hat{E}_a^+) - \\ & - \mathcal{S}_a\,(2\,\hat{E}_a^+ \,\hat{\rho}\, \hat{E}_a^+ - \hat{E}_a^+ \hat{E}_a^+ \hat{\rho} - \hat{\rho}\, \hat{E}_a^+ \hat{E}_a^+) - \\ & - \bar{\mathcal{S}}_a\,(2\,\hat{E}_a^- \,\hat{\rho}\, \hat{E}_a^- - \hat{E}_a^- \hat{E}_a^- \hat{\rho} - \hat{\rho}\, \hat{E}_a^- \hat{E}_a^-)\big].\end{aligned}$$

$$\mathcal{N}_a = [(< n > + \frac{1}{2})\, ch(2\,r_j) - \frac{1}{2}]|_{\omega_j = \omega_a}, \quad \mathcal{S}_a = [(< n > + \frac{1}{2})\, e^{i\,\theta_j}\, sh(2\,r_j)]|_{\omega_j = \omega_a}$$

$$|0>_{sq} = \exp[(\bar{\zeta}\hat{b}^2 - \zeta\,\hat{b}^{+2})/2]\,|0>, \quad \zeta = r\,e^{i\theta}.$$

SUMMARY

- Group theoretical method and the CS technique are naturally used in quantum optics, quantum information theory, condensed matter and so on.
- Search for quantum corrections to the semi-classical dynamics CS in this approach in the general case has not been solved to date.
- One of the main problems here is the inclusion of non-Markovian effects into consideration.
- Possible generalizations of the concept of dynamical symmetries (super-algebras, associative algebras, …) to more complicated and realistic systems also a worthy of special consideration.

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