Dynamical symmetry of two qubits in external fields with dipole – dipole interaction

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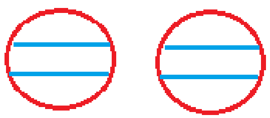
Abstract

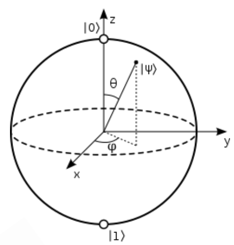
Applications of dynamical groups to the temporal dynamics of multilevel quantum systems in external fields are considered. Using the representation of coherent states (CS), it is shown that the time evolution of the state vector is reduced to the "classical" dynamics of complex parameters of the CS, taking values in the space of cosets of the dynamical group of the Hamiltonian. A particular case of the dynamics of two dipole-dipole (dd) interacting qubits used in the schemes of modern quantum informatics is studied in framework of SU(4) group.

Model and approach



ω1 ω2

Two dd-interacted qubits



$$\hat H(t) = \hat H_0 + \hat V_I(t),\ \hat H_0 = \hbar\omega_1 \hat J_3^{(1)} + \hbar\omega_2 \hat J_3^{(2)},$$

$$V_I(t) = -\frac{\hbar g^{(12)}}{2}\left(J_+^{(1)} J_-^{(2)} + J_-^{(1)} J_+^{(2)}\right) - -\frac{\hbar}{2}\left\{\left(\Omega_1 J_+^{(1)} + \Omega_2 J_+^{(2)}\right)\exp(i\Delta\omega t) + h.c.\right\}.$$

$$G = SU(2)\times SU(2) \subset SU(4),$$

$$|z_1, z_2\rangle = |z_1\rangle \otimes |z_2\rangle, |z_\alpha\rangle = (1+z_\alpha \bar z_\alpha)^{-1/2}\exp(z_\alpha \hat J_+^{(\alpha)})|1/2,\downarrow\rangle,\ \hat J_+ = \begin{pmatrix} 0 & 1\\ 0 & 0\end{pmatrix}.$$

$$|z_1\rangle \otimes |z_2\rangle \Rightarrow |\xi\rangle \equiv |\xi_1,\xi_2,\xi_3\rangle - SU(4)\ \text{coherent state;}$$

$$i\hbar \sum_{\beta=1}^{3} \frac{\partial^2 \ln K(\xi,\bar\xi)}{\partial\bar\xi^\alpha \partial\xi^\beta}\dot\xi^\beta = \frac{\partial \mathcal{H}(\xi,\bar\xi;t)}{\partial\bar\xi^\alpha} \Leftarrow \text{exact equations!}$$

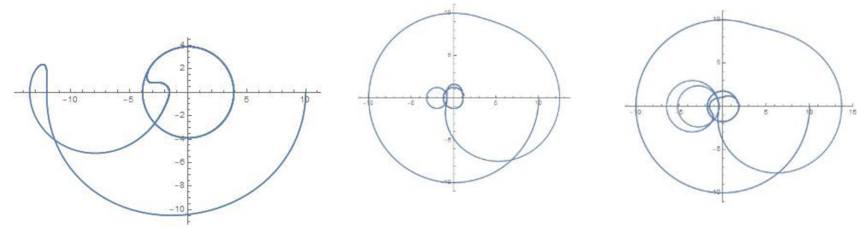
$$\alpha,\beta = 1,2,3.$$

$$\mathcal{H}(\xi,\bar\xi;t) = \langle\xi|\hat H(t)|\xi\rangle,\ K(\xi,\bar\xi) = \left(1+\xi_1\bar\xi_1 + \xi_2\bar\xi_2 + \xi_3\bar\xi_3\right)^{-1}.$$

$$|z_1\rangle \otimes |z_2\rangle = (1+z_1\bar z_1)^{-1/2}\cdot(1+z_2\bar z_2)^{-1/2}\begin{pmatrix} z_1\cdot z_2\\ z_2\\ z_1\\ 1\end{pmatrix}, \text{(no qubits entanglement!)}$$

$$|\xi_1,\xi_2,\xi_3\rangle = (1+\xi_1\bar\xi_1 + \xi_2\bar\xi_2 + \xi_3\bar\xi_3)^{-1/2}\begin{pmatrix}\xi_3\\ \xi_2\\ \xi_1\\ 1\end{pmatrix},\ (\xi_3 \neq \xi_1\cdot\xi_2 - \text{entanglement!})$$

Numerical calculations



Trajectories in complex planes ξ_1, ξ_2, ξ_3, respectively.
$\xi_3(0) = \xi_1(0)\,\xi_2(0)$

References

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