Exploring the Universe around the Big Bang

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Abstract— a model of the early universe at times close to the Big Bang is proposed where the Friedman-Robertson-Walker model does not work. It is based on an asymptotic study of the Einstein-Maxwell equations at short distances. It is shown that under certain conditions their solutions acquire spherical symmetry. A criterion for isotropization of solutions of the Einstein-Maxwell equations is formulated. Solutions of the Maxwell-Einstein equations allow to find a space-time metric. At short distances, the metric has a singular Schwarzschild form, where the mass of matter in the entire universe plays the role of mass. On the periphery, the metric is determined from Maxwell's equations in a curved space-time. It is shown that the singularity is shielded by this peripheral solution. The results obtained make it possible to clarify the details of phenomena in the field of modern cosmology, as well as the theory of electromagnetism and gravity.

Keywords: Maxwell-Einstein equations, Friedman-Robertson-Walker model, evolution of the early universe, focusing of an electromagnetic wave, ray bending, isotropization, asymptotic.

INTRODUCTION

The dynamics of a homogeneous and isotropic universe is described using the Einstein equations of gravity in the framework of the Friedman–Robertson–Walker model [1] (see section 2). According to it, the beginning of the universe was laid by the so-called Big Bang, which is understood as a dense, localized state of matter and radiation, marked in the model as a singularity at time $t = 0$. The origins of this singularity, as well as the prehistory of the universe in the moments preceding it, are not considered.

The limitation of the theory is that while describing the state of matter in the universe as homogeneous and isotropic, which corresponds to modern observations, it does not allow these ideas to be extended to times immediately close to the Big Bang, about which various assumptions are permissible. This is due to the fact that the state of matter in the FRW model is described using conservation laws, the fulfillment of which requires the thermalization of matter (radiation), which takes time and is problematic due to the presence of particle horizons. One way to solve this problem is the assumption of isotropy of the universe at the earliest times, before thermalization occurs. Clarifying the situation is aided by the solutions of Einstein's equations and the equations of dynamics that determine the behavior of matter. Such equations in the case of radiation dominance are Maxwell's equations. Sections 3 and 4 of the article are devoted to solving this problem. Make a reservation right now that we are not talking about the exact solution of the M-E equations, but only about the study of the asymptotic of the solution in the vicinity of a Big Bang.

Section 5 is devoted to finding the criterion for isotropization of the solution of the M-E equations due to the effect of bending of the light rays.

Section 6 is devoted to discussing the results.

THE TRADITIONAL PICTURE OF THE DYNAMICS OF THE UNIVERSE

The dynamics of a homogeneous and isotropic universe is described within the framework of the Friedman–Robertson–Walker (FRW) model, the metric of which is given by the expression [1]

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] \quad (1)$$

k = (-1, 0, 1) – curvature index determined by the type of Friedman model (open, flat, closed) [1], $a(t)$ is a model parameter (scalar factor) determined from Einstein's equations.

The evolution of the universe in the FRW model is completely determined by the parameter $a(t)$. Its behavior as a function of time depends on the behavior of density of matter, radiation, and vacuum energy as a function of a. All these quantities are determined based on the first law of thermodynamics, which expresses the law of conservation of energy [1]:

$$\frac{d}{dt}\left[\rho_c(t)a^3(t)\right] = -p_c(t)\frac{d}{dt}\left[a^3(t)\right] \quad (2)$$

Here ρ_c – is energy density, p_c – is pressure of the corresponding component. From Einstein's equations follows the equation for $a(t)$ [1] ($k = 0$):

$$\dot{a}^2 - \frac{8\pi\rho}{3}a^2 = 0 \quad (3)$$

ρ - total energy density; the dot means the derivative with respect to time t. Equation (3) can be solved for each component separately, which makes it possible to determine the dependence $a_c(t)$ on t for cases of dominance of each component: $c = m$ (dust matter), $c = r$ (radiation), $c = v$ (vacuum) [1]:

$$a_m(t) = \left(\frac{t}{t_0}\right)^{2/3}, a_r(t) = \left(\frac{t}{t_0}\right)^{1/2}, a_v(t) = e^{H(t-t_0)} \quad (4)$$

Here $a(t_0) = 1$ – normalization condition, and $H^2 = \Lambda/3$, Λ – is the cosmological constant; t_0 – is the moment in time at which the observation is performed [1]. Cases of dominance of matter and radiation are characterized by a singularity at $t = 0$, corresponding to the Big Bang and characterized by an infinite energy density of both matter and radiation.

The FRW model has proven to be effective in explaining phenomena in a sufficiently developed universe, describing it as homogeneous and isotropic. However, the existence of so-called particle horizons does not allow these properties to be definitively extended to the universe at the moment of its birth. Therefore, it makes sense to consider this problem without resorting to equation (2), which assumes that the processes of matter thermalization, or rather radiation, as in the case of the early universe, have concluded with the formation of an equilibrium state. In other words, instead of solving Einstein's gravity equations, whose right-hand side contains the energy-momentum tensor of equilibrium radiation, one should use the Maxwell-Einstein equations, with the energy-momentum tensor of the initial electromagnetic field.

SETTING THE TASK

Let us choose coupled Einstein's equation of gravity and Maxwell's equations of electromagnetic field in vacuum as basic ones [2, 3]:

$$\begin{aligned} & R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik}; \\ & F^{ik}_{,k} + \Gamma^l_{kl} F^{ik} = 0; \\ & R_{ik} = \Gamma^l_{ik,l} - \Gamma^l_{il,k} + \Gamma^l_{ik}\Gamma^m_{lm} - \Gamma^m_{il}\Gamma^l_{km} \\ & T_{ik} = \frac{1}{4\pi}\left(- F_{il}F^l_k + \frac{1}{4} g_{ik} F_{lm} F^{lm} \right) \end{aligned} \tag{5}$$

Here $R = R^i_i$ – is a trace of Ricci's tensor R^i_k; g_{ik} – the metric tensor; T_{ik} and F^{ik} – tensors of energy-momentum and electromagnetic one; Γ^i_{kl} – Christoffel's symbols; c – light speed in vacuum, G – the gravitation constant; indices i, k, l take values 0, 1, 2, 3; repeated indices mean summation; comma means usual, i.e., non-covariant derivative [2].

A lot of attention has been paid to the study of solutions to the M-E equations, and a number of important results have been obtained [4, 5], revealing the mathematical side of the problem. In this article, the main attention is paid to the physical side of the issue, when it is not the strict formulas that are important, but the physics of phenomena.

Strictly speaking, the problem does not have spherical symmetry. However, due to the properties of electromagnetic and gravitational fields (gravitational bending of the light rays, nonlinear diffraction), the angle dependence in the distribution of field sources is weakened at short distances. Therefore, we will look for an interval in the form (this assumption will be justified below):

$$\begin{aligned} & ds^2 = g_{00} c^2 dt^2 + g_{11} dr^2 - r^2\left(d\theta^2 + sin^2\theta \cdot d\varphi^2\right) \\ & g_{00} = e^{\nu}, g_{11} = -e^{\lambda} \end{aligned} \tag{6}$$

$\nu = \nu(t, r)$, $\lambda = \lambda(t, r)$; t – time; r, θ, φ – spherical coordinates. This type of interval is applicable to problems with spherical symmetry (like in the Schwarzschild problem) and is dictated by the assumption of approximate spherical symmetry of our problem for small r, which, as shown below, under certain conditions becomes accurate when the size of the universe tends to zero.

We will look for solutions to equations (5) corresponding to the presence at infinity of a spherical electromagnetic wave (SEMW) with frequency ω and angular momentum $\vec{L}$. Let's choose the OZ axis of the coordinate system in the direction perpendicular to $\vec{L}$. This will si mplify further calculations.

It leads from (5) (second pair of Maxwell equations):

$$\frac{\partial}{\partial x^{\beta}}\left(\sqrt{-g}F^{\gamma\beta}\right)-\frac{\partial}{\partial x^{0}}\left(\sqrt{-g}F^{0\gamma}\right)=0$$
$$\frac{\partial}{\partial x^{\beta}}\left(\sqrt{-g}F^{0\beta}\right)=0;\ \beta,\gamma=1,2,3; \qquad (7)$$
$$\sqrt{-g}=e^{\frac{\lambda+\nu}{2}}r^{2}\sin\theta$$

Here γ =1, 2 correspond to the SEMW of the TM - type, and γ=3– for the SEMW of the TE – type [2] . Below we restrict ourselves to the case of the TM - type for nonzero components F_{ik}, assuming that nonzero components of vector-potential A_i are only A_1, A_2 (in the Hamilton calibration A_0= 0):

$$F_{01}=\frac{\partial A_1}{\partial x^0},F_{12}=\frac{\partial A_2}{\partial x^1}-\frac{\partial A_1}{\partial x^2},F_{02}=\frac{\partial A_2}{\partial x^0} \quad (8)$$

This ensures the validity of the first pair of the Maxwell equations.

We will impose an additional condition: $\lambda=-\nu=\alpha(r,t)$. Substituting (8) into (7), we obtain two equations for the components A_1 and A_2

$$e^{-\alpha(r,t)}\sin\theta\frac{\partial}{c\partial r}\left[r^2\frac{\partial A_1}{\partial t}\right]+\frac{\partial}{c\partial\theta}\left[\sin\theta\frac{\partial A_2}{\partial t}\right]=0;$$
$$\frac{\partial}{\partial\theta}\left[\sin\theta\left(\frac{\partial A_1}{\partial\theta}-\frac{\partial A_2}{\partial r}\right)\right]-e^{\alpha(r,t)}\frac{r^2}{c^2}\sin\theta\frac{\partial^2 A_1}{\partial t^2}=0 \qquad (9)$$

The first equation in (7) follows from (9). If we take a derivative of the first equation in (9) on r and the second – on ct, then we exclude A_2 from the equations. Put F_{01}= $\Psi(r,t)\cdot\Phi(\theta)$, then separating variables, we receive equations for $\Psi(r,t)$ and $\Phi(\theta)$:

$$\frac{d^2\Phi}{d\theta^2}+ctg\theta\frac{d\Phi}{d\theta}+l(l+1)\Phi=0;$$
$$\frac{\partial}{\partial r}\left[e^{-\alpha(r,t)}\frac{\partial}{\partial r}\left(r^2\Psi\right)\right]-e^{\alpha(r,t)}\frac{\partial^2}{c^2\partial t^2}(r^2\Psi)-l(l+1)\Psi=0 \qquad (10)$$

It leads from (10) that $\Phi(\cos\theta)=P_l(\cos\theta)$, where $P_l(\cos\theta)$ is the Legendre polynomial, and l is a nonnegative integer.

The energy-momentum tensor's components T_k^i may be expressed by the components of the metric tensor g_k^i with the help of Einstein's equation of gravity [2]:

$$\frac{8\pi G}{c^4}T_k^i=G_k^i$$
$$G_k^i=R_k^i-\frac{1}{2}\delta_k^i R \qquad (11)$$

Here G_k^i –is the Einstein tensor; δ_k^i is the unit 4-tensor. The details of calculations one can find in [2], for example. The expressions for G_k^i look as follows [2]:

$$
\begin{aligned}
G_0^0 &= -e^{-\alpha}\left(\frac{1}{r^2}-\frac{\alpha'}{r}\right)+\frac{1}{r^2};\\
G_1^1 &= -e^{-\alpha}\left(\frac{1}{r^2}-\frac{\alpha'}{r}\right)+\frac{1}{r^2};\\
G_2^2 &= \frac{1}{2}e^{-\alpha}\left[\alpha''-(\alpha')^2+\frac{2\alpha'}{r}\right]+\frac{1}{2}e^{\alpha}\left(\ddot{\alpha}+\dot{\alpha}^2\right);\\
G_3^3 &= \frac{1}{2}e^{-\alpha}\left[\alpha''-(\alpha')^2+\frac{2\alpha'}{r}\right]+\frac{1}{2}e^{\alpha}\left(\ddot{\alpha}+\dot{\alpha}^2\right);\\
G_0^1 &= -e^{-\alpha}\frac{\dot{\alpha}}{r};\, G_0^2=0;\, G_1^2=0
\end{aligned}
\qquad (12)
$$

We also express components of the energy-momentum tensor T_k^i from the Maxwell equation's solutions [2]:

$$
\begin{aligned}
T_0^0 &= \frac{1}{8\pi}\left\{\Psi^2[P_l]^2+\frac{[P_l^1]^2}{[l(l+1)]^2}\left[\frac{1}{r^2}e^{-\alpha}\left(\frac{\partial}{\partial r}r^2\Psi\right)^2+r^2e^{\alpha}\left(\frac{\partial\Psi}{\partial x^0}\right)^2\right]\right\};\\
T_1^1 &= \frac{1}{8\pi}\left\{\Psi^2[P_l]^2-\frac{[P_l^1]^2}{[l(l+1)]^2}\left[\frac{1}{r^2}e^{-\alpha}\left(\frac{\partial}{\partial r}r^2\Psi\right)^2+r^2e^{\alpha}\left(\frac{\partial\Psi}{\partial x^0}\right)^2\right]\right\};\\
T_2^2 &= \frac{1}{8\pi}\left\{-\Psi^2[P_l]^2+\frac{[P_l^1]^2}{[l(l+1)]^2}\left[\frac{1}{r^2}e^{-\alpha}\left(\frac{\partial}{\partial r}r^2\Psi\right)^2-r^2e^{\alpha}\left(\frac{\partial\Psi}{\partial x^0}\right)^2\right]\right\};\\
T_3^3 &= \frac{1}{8\pi}\left\{-\Psi^2[P_l]^2-\frac{[P_l^1]^2}{[l(l+1)]^2}\left[\frac{1}{r^2}e^{-\alpha}\left(\frac{\partial}{\partial r}r^2\Psi\right)^2-r^2e^{\alpha}\left(\frac{\partial\Psi}{\partial x^0}\right)^2\right]\right\};\\
T_0^1 &= -\frac{1}{4\pi}\frac{e^{-\alpha}}{[l(l+1)]^2}\frac{\partial\Psi}{\partial x^0}\frac{\partial}{\partial r}\left(r^2\Psi\right)[P_l^1]^2;\\
T_0^2 &= \frac{1}{4\pi}\frac{1}{l(l+1)}\Psi\frac{\partial\Psi}{\partial x^0}P_lP_l^1;\\
T_1^2 &= \frac{1}{4\pi}\frac{1}{l(l+1)}\frac{\Psi}{r^2}\frac{\partial}{\partial r}\left(r^2\Psi\right)P_lP_l^1;
\end{aligned}
\qquad (13)
$$

Here $P_l=P_l(\cos\theta), P_l^1=P_l^1(\cos\theta)$; P_l^1 are the associated Legendre polynomials.

The Einstein equations (11) will be valid if the energy-momentum tensor T_k^i in the left side will be averaged over the angle θ and time t. The possibility of this will be founded below. In particular, we will show that as the size of the universe decreases its state becomes isotropic in the vicinity of the Big Bang.

If one subtracts the second equation from the first and the fourth from the third in (13) and equates these results to the results of the same operation in (12), he comes to the following equations:

$$\left\langle e^{-\alpha}\left(\frac{\partial}{\partial r}r^2\Psi\right)^2+e^{\alpha}\left(\frac{\partial}{\partial x^0}r^2\Psi\right)^2\right\rangle_{t,\theta}=0;$$
$$\left\langle e^{-\alpha}\left(\frac{\partial}{\partial r}r^2\Psi\right)^2-e^{\alpha}\left(\frac{\partial}{\partial x^0}r^2\Psi\right)^2\right\rangle_{t,\theta}=0; \quad (14)$$

$\langle\ \rangle_{t,\theta}$ - denotes the operation of averaging over time and angle θ. As will be shown below, these equalities apply to solutions of Einstein-Maxwell equations of different types.

From the second equation in (14), one can find e^{α}

$$e^{\alpha}=\pm\frac{\partial f}{\partial r}\left(\frac{\partial f}{\partial x^0}\right)^{-1}; (15)$$

Solutions of (10), corresponding to the first equation in (14), can be treated in pseudo-Euclidean space, which metric follows from the Minkowski metric by substituting the time coordinate x^0 to the "imagine time" coordinate $y^0 = ix^0$ in pseudo-Euclidean space. At the same time, one can introduce the pseudo-Euclidean action Λ, which is connected with the action S in Minkowski space as follows: $\Lambda = iS$, $i = (-1)^{1/2}$. It is known [6] that localized solutions of the Euclidean field equations with the finite pseudo-Euclidean action are instantons. Instantons of classical field equations in Minkowski space describe in the quasi-classical limit, tunneling between degenerate classical states, which roles play the convergent and divergent SEMW.

This procedure turns the second equation in (10) from the hyperbolic to the elliptic type. If one supposes its finiteness at $r \to \infty$, then he receives from the first equation (14) an expression for e^{α} equivalent to (15):

$$e^{\alpha}=\pm\frac{\partial f}{\partial r}\left(\frac{\partial f}{\partial y^0}\right)^{-1} \quad (16)$$

FINDING THE METRICS

In what follows, we will assume that α does not depend on t: $\alpha=\alpha(r)$ - this will greatly simplify calculation. The simplest way to find the metric is to substitute expressions (15) or (16) for e^{α} into the second equation (10) and then solve it. These solutions have the form

$$f^{I,II}(r,t)=\begin{Bmatrix} f_0^I\, exp(\pm ir_c/r) \\ f_0^{II}\, exp(\pm r_c/r)\end{Bmatrix} exp\left[e^{i\omega(t-t_0)}\right]$$
$$r_c=\frac{c}{\omega}l(l+1) \quad (17)$$

$f_0^{I,II}$ are constants. Then, from (15) and (16), we find in both cases at $\omega(t-t_0)<<1$

$$g_{00}=-g_{11}^{-1}=\left(\frac{r}{r_c}\right)^2 l(l+1) (18)$$

which does not contain the gravitational constant G.

The expression for the metric (18) can be used at distances, where the gravitational component of the wave field is small compared to the electromagnetic one.

For $r \to 0$ near the field singularity, where the gravitational component dominates over the electromagnetic one, we obtain an expression for the metric from Einstein's equations of gravity. Then, substituting the expression found for e^{α} into the second equation (10), we find an expression for the field $f(r,t)$. In this case, we obtain solutions of two types, similar to (17).

To find the metric, we solve equations (12) together with (13), and for different types of solutions we should use the equations for T_0^0 or T_1^1 (12) and (13) or for T_2^2 or T_3^3 (12) and (13). Let's consider the first pair of equations. Then, after performing the necessary averaging operations over θ and t, we obtain a differential equation for determining $\alpha(r)$, the solution of which has the form

$$\begin{gathered} g_{00} = e^{-\alpha} = 1 - \frac{r_w}{r}, g_{11} = -g_{00}^{-1} \\ r_w = \frac{2GW_r}{c^4}, W_r = \frac{1}{2l+1}\int_r^R \frac{\Psi^2}{8\pi} 4\pi r^2 dr \end{gathered} \tag{19}$$

R has a meaning of the universe size. Here, W_r represents the energy associated with the component of the wave field F_{01}. Formula (19) takes the Schwarzschild form if we introduce the concept of the mass $M = W_r/c^2$ of matter in the form of radiation at the earliest stage of the evolution of the universe according to the Mach principle.

Let us find the condition under which the equations for T_2^2 and T_3^3 (12) and (13) will be satisfied by the solution (19). To do this, we integrate the corresponding equations (11). As a result, we get (the last expression is valid when $r \to 0$)

$$\begin{gathered} y' = \frac{2G}{c^4(2l+1)}\frac{1}{r^2}\int_r^R \frac{\Psi^2}{8\pi} 4\pi r^2 dr; y = e^{-\alpha} \\ y \approx 1 - \frac{2G}{c^4(2l+1)}\frac{1}{r}\int_r^R \frac{\Psi^2}{8\pi} 4\pi r^2 dr \end{gathered} \tag{20}$$

which coincides with (19), provided that the integral representing the total energy of the matter of the universe in the form of radiation is finite.

The ME equations for T_0^2, T_1^2 after averaging over θ are satisfied due to the orthogonality of P_l and P_l^1. The equation for T_0^1 is satisfied after averaging over t.

The expression for $f(r,t)$ is obtained by substituting the expression for g_{00} (15) or (16) into equations (14). Then solving the second equation in (10), we have $\left(f(r,t) = r^2\Psi(r,t)\right)$

$$\begin{gathered} f^I(r,t) = f_0^I \left|\frac{r}{r_{w0}} - 1\right|^{\pm i\frac{\omega r_{w0}}{c}} e^{i\omega\left(t \pm \frac{r}{c}\right) - \frac{r_c}{r}} \\ f^{II}(r,t) = f_0^{II} \left|\frac{r}{r_{w0}} - 1\right|^{\pm\frac{\omega r_{w0}}{c}} e^{\pm\frac{\omega r}{c} - \frac{r_c}{r} + i\omega t} \end{gathered} \tag{21}$$

f_0^I, f_0^{II} are constants, $r_c = \dfrac{cl(l+1)}{\omega}, r_{w0} = r_w(r=0)$. If one substitutes (21) into (19) one receives

$\dfrac{dr_w}{dr} \xrightarrow[r \to 0]{} 0$ so r_{w0} can be named the "gravitational radius" of the matter in the universe.

ISOTROPIZATION OF THE GRAVITATIONAL FIELD SOURCE

The trajectory of a beam of light propagating in its own gravitational field with metric (6), taking into account (19), is determined from the eikonal equation ϕ [2]:

$$g^{ik}\frac{\partial\phi}{\partial x^i}\frac{\partial\phi}{\partial x^k}=0 \qquad (22)$$
$$\phi(r,t,\varphi)=-\omega t+m\varphi+\phi_r(r)$$

$\phi_r(r)$ is the radial part of an eikonal, and ω and m are the constants. Since metric (19) formally coincides with the Schwarzschild metric [2], we can use the well-known results [2] to calculate the beam deflection angle. The angle of deviation of the trajectory from the straight line on the way from $r=\infty$ to $r=\rho$ is equal to

$$\Delta\varphi=\pi+\frac{2r_W}{\rho},\ \rho=\frac{cm}{\omega} \qquad (23)$$

Here, instead of the constant m, the impact distance ρ is introduced.

The current direction of the beam associated with the section of the converging wave front with the coordinates $r,\ \theta,$ and φ is determined by the gradient of the wave phase $\Phi(r,\varphi)=-kr+m\varphi$, k – is a wave number:

$$\nabla\Phi=-k\vec{r}_0+\frac{m}{\tilde{r}}\vec{\varphi}_0 \qquad (24)$$

Here $\vec{r}_0,\vec{\varphi}_0$ are spherical unit vectors. Let's express the impact distance ρ in terms of the task parameters:

$$\rho=\tilde{r}\,Sin\,\gamma,\ tg\,\gamma=\frac{m}{krSin\ \theta}$$
$$Sin\,\gamma=\frac{tg\,\gamma}{\sqrt{1+tg^2\gamma}}=\frac{m}{\sqrt{m^2+(kr)^2\,Sin^2\theta}} \qquad (25)$$

$\tilde{r}=rSin\theta$, here $\tilde{r}$ is the length of projection of $\vec{r}$ on the beam trajectory plane, and γ is the angle between $\nabla\Phi$ and $\vec{r}_0$. From (25) we receive the expression for ρ:

$$\rho=\frac{m}{k}\frac{1}{\sqrt{1+\dfrac{m^2}{(krSin\theta)^2}}}\xrightarrow[r\to\infty]{}\frac{m}{k} \qquad (26)$$

Based on (26) and (23), it is possible to formulate a condition for the overlap of neighboring inhomogeneities in the angular distribution of gravitational field sources, which has the order $\frac{\pi}{l+1}$

$$2\frac{kr_W}{m}>\frac{\pi}{l+1} \qquad (27)$$

or if we introduce wavelength $\lambda=\frac{2\pi}{k}$

$$4\frac{r_w}{\lambda} > \frac{m}{l+1} \quad (28)$$

which can be performed in spatial and temporal proximity to the Big Bang. This, in turn, confirms the hypothesis of isotropization of the field distribution over small distances and allows us to perform averaging over θ in the equations (12) and (13).

The criterion (28) is obtained under condition $r_w \approx const$ which is valid at small r as shown above what is allowable because the main ray deflection originates just at these distances r. The contribution of large distances will give corrections to the above expressions, which will only strengthen criterion (28).

If we perform similar calculations for metric (18), we get the expression for the angle $\varphi(r)$ and deflection of the light beam $\Delta\varphi$

$$\begin{aligned} \varphi(r) &= \frac{1}{\sqrt{1-(\rho/r_c)^2}}\left(1-\frac{\rho}{r}\right) \\ \Delta\varphi &= \frac{1}{\sqrt{1-(\rho/r_c)^2}} \end{aligned} \quad (29)$$

where ρ – is an impact distance valid for $\rho < r_c$. Since ρ and r_c are proportional to the wavelength λ, this condition is independent of λ and leads to obvious inequality $l(l+1) > m^2$ if we use the limit expression (26) for ρ. The behavior of $\varphi(r)$ is characterized for the wave self-focusing.

DISCUSSION

The study of the Maxwell-Einstein equations performed in this article differs from the search for exact solutions of these equations [3, 4] in its applied nature. As the authors themselves point out [3, 4], all exact solutions of the M-E equations suffer from a common disadvantage, experiencing difficulties with interpretation. This is not surprising, because the purpose of obtaining them was themselves and not using them in physical tasks. In contrast to obtaining exact solutions to the M-E equations, the purpose of the present research was to study specific problems of the cosmology and electrodynamics: the evolution of the early universe and the focusing of spherical electromagnetic waves.

This formulation of the question made it possible, instead of searching for exact solutions to the M-E equations, to limit ourselves in studying their asymptotic for small r, which greatly simplified the task and at the same time expanded the scope of application of the results obtained.

The study of the M-E equations performed above makes it possible to evaluate the asymptotic of a wide class of exact solutions of the M-E equations at short distances and draw conclusions about the nature of physical processes in which the conditions for the specified asymptotic are valid.

One should say about the choice of an asymptotic metric in the form of (6). In principle, instead of (6), one could choose any spherically symmetric metric from those that allow exact solutions of the M-E equations [4]. All of them allow us to obtain results similar to those given above, since the phenomenon of gravitational bending is universal. Possible differences relate to the type of isotropization criterion (28). Therefore, the choice of metric (6) is due only to the fact that it is studied better than others, which made it possible to use the known results [2].

One possible application of the obtained results is the theory of the early universe [1]. First of all, this pertains to phenomena in the vicinity of the so-called Big Bang, where the predictions of the Friedman-Robertson-Walker model are limited. This is due to the presence of the so-called particle horizons [7], the existence of which does not allow certain assumptions about the initial state of the universe. The presented results can be interpreted in favor of R. Penrose's arguments about the homogeneous and isotropic state of the universe at the moment of its origin [8].

Another phenomenon for which the obtained results can be used is the focusing of a spherical electromagnetic wave. Let's look at this issue in more detail. Within the framework of traditional Maxwell's electrodynamics, it cannot be solved, since it leads to the appearance of a singularity at the point of the assumed focus at $r = 0$. There are known attempts to solve it using fictitious sources [9]. To solve this issue within the framework of physical theory, it should be taken into account that the electromagnetic wave field is large near the focus point $r = 0$, and this suggests the need to take into account its influence on the curvature of space-time, i.e., to take into account the nonlinear action of light on itself. The presence of two solutions of equations M-E (17) allows us to consider the problem of converting a converging light wave into a diverging one as a problem of stitching the two solutions at the boundary of their existence. Let's assume that this boundary is located at $r = r_c$ and $r_c >> r_w$. We write down a general expression for the spatial part of the solution in the areas $r < r_c$ and $r > r_c$:

$$f(r) \sim exp\left(i\frac{r_c}{r}\right) + Re\,xp\left(-i\frac{r_c}{r}\right), r > r_c$$
$$f(r) \sim A\,exp\left(\frac{r_c}{r}\right) + B\,exp\left(-\frac{r_c}{r}\right), r < r_c \quad (30)$$

Stitching together these expressions for $r = r_c$, we find the values of the reflection coefficient R and the coefficient B with a decreasing exponent for $r \to 0$ under the additional condition $f(r = 0) = 0$, providing the absence of an electromagnetic field singularity. The last condition immediately allows us to find $A = 0$. Here is the final result:

$$R = -ie^{2i}$$
$$B = (1-i)e^{1+i} \quad (31)$$

This means complete transformation of a converging wave into a diverging one, due to the complete reflection of the incident wave: $|R| = 1$. In cosmology, the obtained result is realized as an analog of the principle of cosmic censorship, prohibiting access to the singularity of the metric (19). In electrodynamics this eliminates the singularity at the focal point of the spherical electromagnetic wave.

In conclusion, we will say a few words about the features of the approach used in this work to the study of the Einstein-Maxwell equations.

A metric of the form (6) is usually used in the study of stationary (static) solutions of EM equations, for example, in the study of the gravitational field in the vicinity of a point charge [2, 10]. In this paper, it is shown that such a metric (on average) can also be used for non-stationary tasks.

Non-singular solutions of the ME equations were studied, for example, in [11]. The existence of such solutions contradicts the well-known singularity theorems [7]. In this paper, singularities are present, but access to them is impossible due to the presence of non-oscillating solutions to the ME equations, which implement a peculiar principle of cosmological censorship.

It was already mentioned above that the FRW model is inapplicable at the earliest stages of the development of the universe near the Big Bang due to insufficient time for thermalization of matter (radiation). This is one of the problems faced by the standard cosmological model of the universe (ΛDCM), which includes the FRW model. Attempts to go beyond this model are going in two ways: theoretical; see [12, 13], e.g., and experimental which are stimulated by astronomical observations carried out using the James Webb Space Telescope (JWST) and the Dark Energy Spectroscopic Instrument (DESI); see, e.g., [14-19].

CONCLUSIONS

Based on an asymptotic study of the Einstein-Maxwell equations at small distances, it is shown in this article that, under certain conditions, the solutions of these equations acquire spherical symmetry, which allows further progress in the study of phenomena in the field of modern cosmology, as well as the theory of electromagnetism and gravity. In cosmology, the results obtained make it possible to shed light on phenomena in the immediate vicinity of the Big Bang. In electrodynamics, the process of focusing spherical waves has been studied, and it has been shown that the influence of the wave's own gravitational field can no longer be ignored near the focus. In the theory of gravity, the asymptotic (at $r \to 0$) behavior of solutions of the Maxwell-Einstein equations is studied, which is common to a wide class of solutions.

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