Research of dynamics of the ensemble-averaged velocity in the "corrugated waveguide" system with an oscillating boundary

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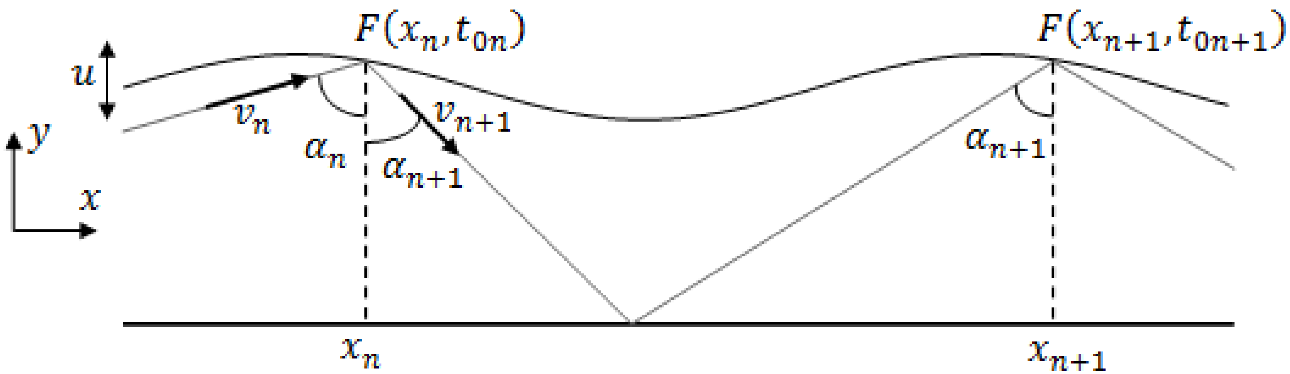
“Corrugated waveguide" system with an oscillating boundary

Consider the particle with a certain mass that moves between two walls, successively hitting and bouncing off each of them.

Top wall equation:

$$y_1 = F(x,t) = b\cos kx + a\cos wt + h;$$



Equations describing the system in dimensionless form:

$$\varphi_{n+1} = \varphi_n - A\frac{\Omega_{n_x}}{\Omega_{n_y}}; \qquad \alpha_{n+1} = arctan\left[-\frac{\Omega_{n_x}}{\Omega_{n_y}}\right];$$

$$\Omega_{n+1} = \sqrt{\Omega_{n+1_x}{}^2 + \Omega_{n+1_y}{}^2}; \quad \psi_{n+1} = \psi_n - \frac{1}{\Omega_{n_y}};$$

$$\Omega_{n+1_x} = \Omega_n \sin(\alpha_n + 2\gamma) - 2\gamma u;$$

$$\Omega_{n+1_y} = -\Omega_n \cos(\alpha_n + 2\gamma) + 2u;$$

$$\gamma = -C\sin\varphi_n; \qquad u = -B\sin\psi_n;$$

$\varphi_n = kx_n;$ – dimensionless coordinate

$\psi_n = wt_{0n};$ – dimensionless time

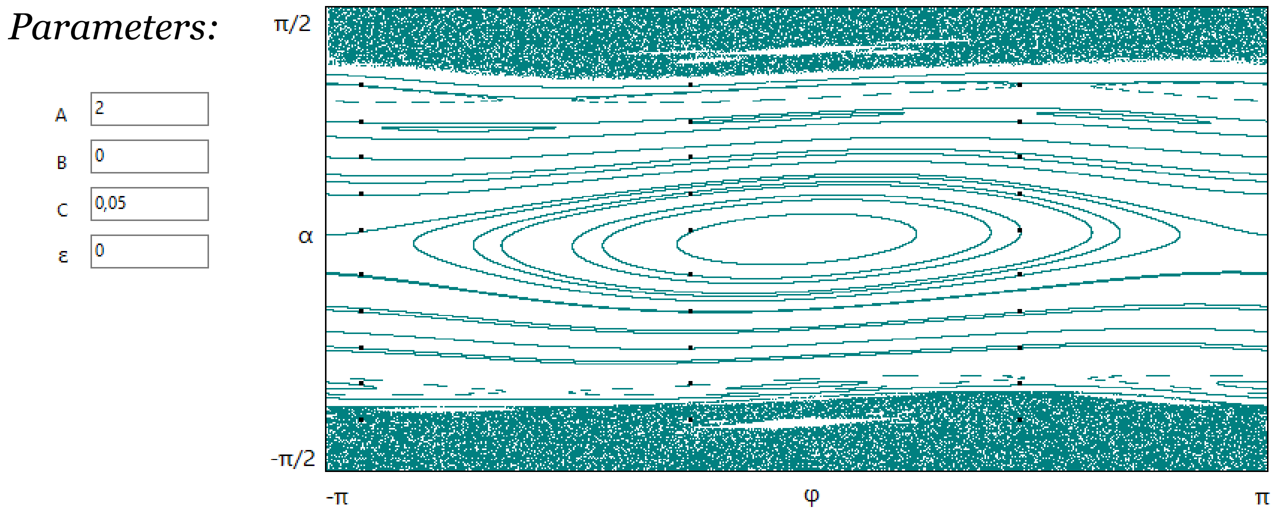
$\Omega_{n_{x,y}} = \dfrac{v_{n_{x,y}}}{2hw};$ – dimensionless velocity

$A = 2hk;$ – dimensionless distance between walls

$B = \dfrac{a}{h};$ – dimensionless amplitude of the wall oscillation

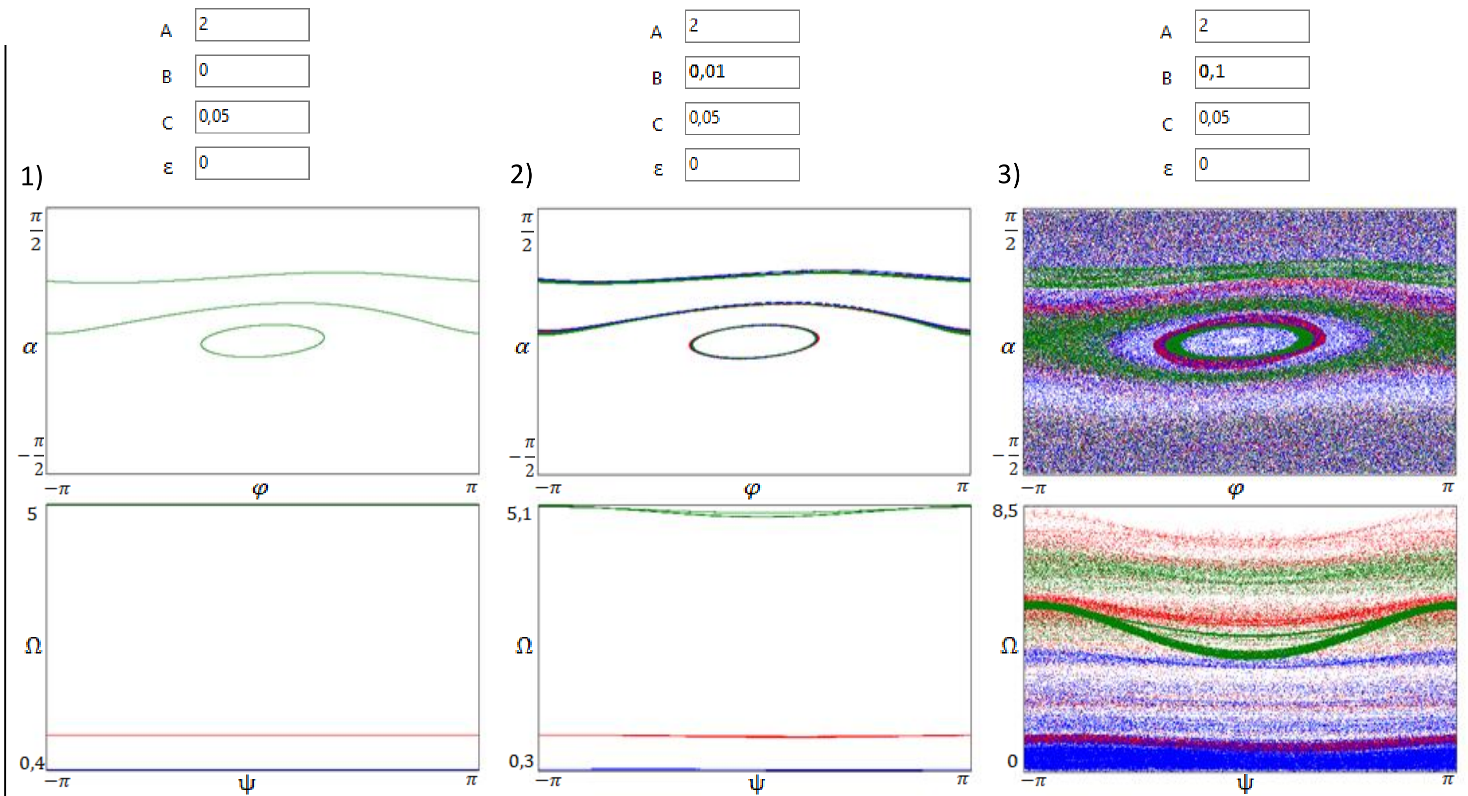
$C = kb;$ – dimensionless amplitude of the wall corrugation

Phase portraits in a conservative situation



Initial conditions grid:

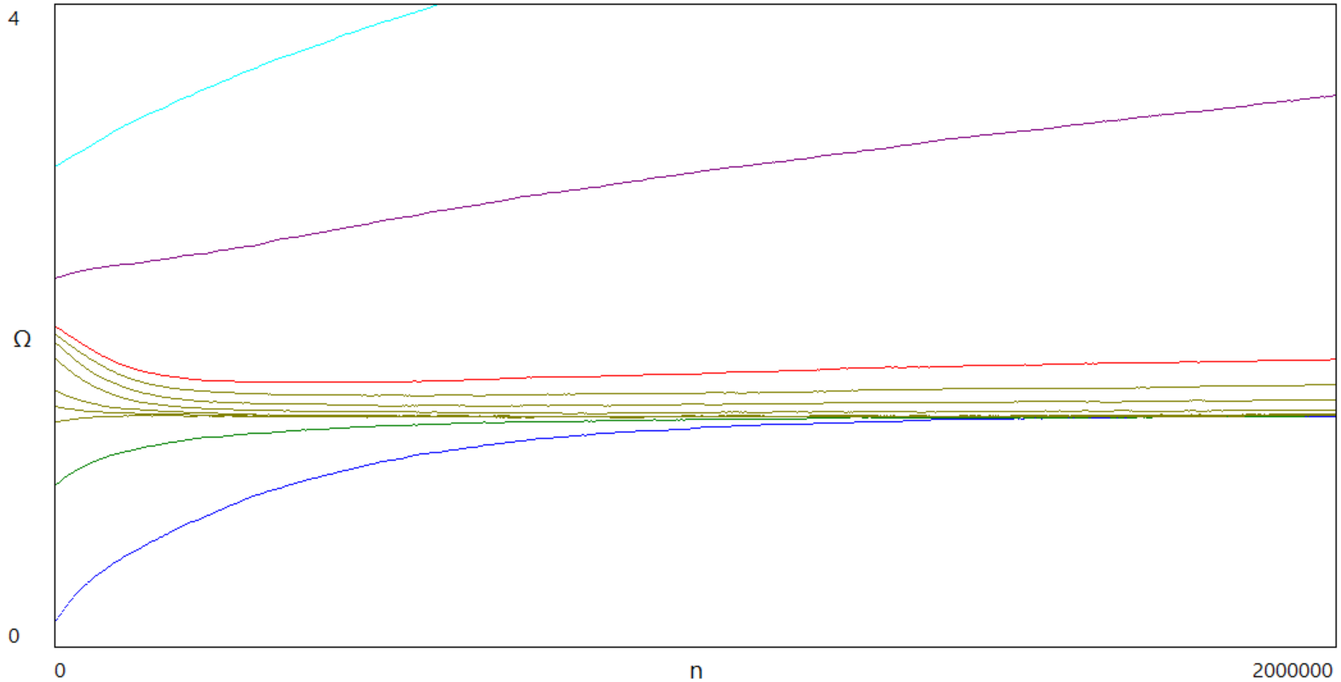
α0	φ0	Ω0	ψ0	
0,06	0	1	4,14	Red
0,14	0	1	4,14	Red
0,25	0	1	4,14	Red
0,06	0	0,4	4,14	Blue
0,14	0	0,4	4,14	Blue
0,25	0	0,4	4,14	Blue
0,06	0	5	4,14	Green
0,14	0	5	4,14	Green
0,25	0	5	4,14	Green



Plots of dependence of the ensemble-averaged velocity on the number of iterations

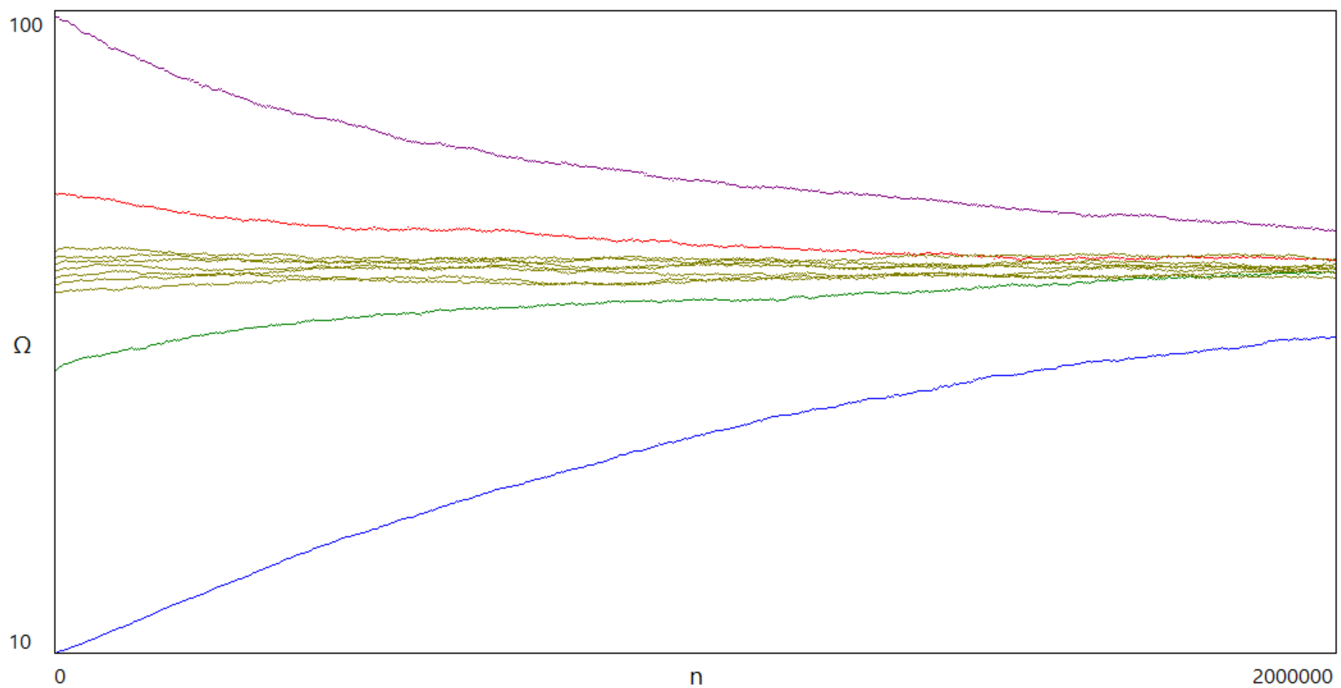
A: 2
B: 0,01
C: 0,05
ε: 0

Blue	0.1
Green	1.0
Red	2.0
Purple	2.3
Light blue	3.0
Yellow	1.4 - 2.0

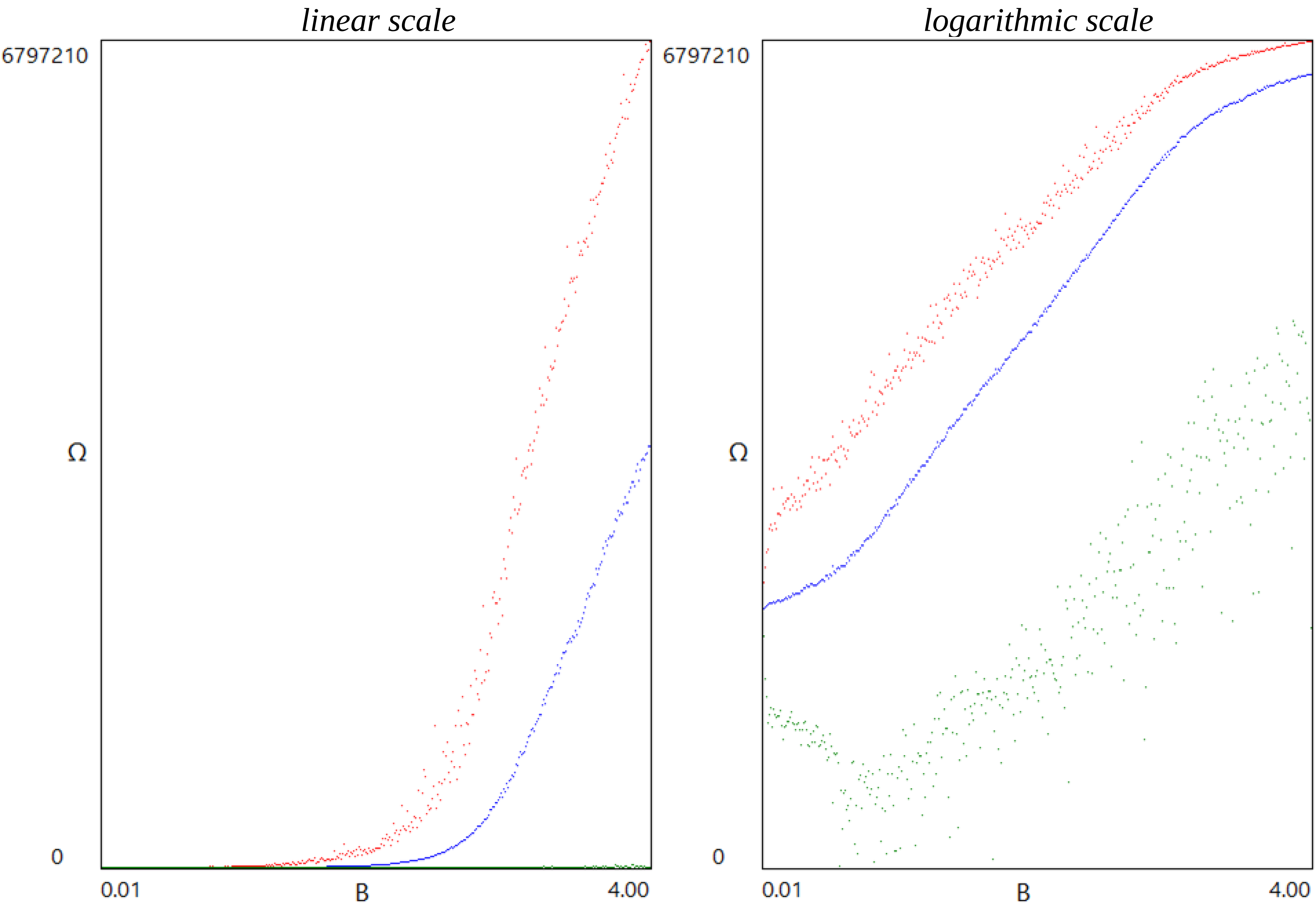


A: 2
B: 0,1
C: 0,05
ε: 0

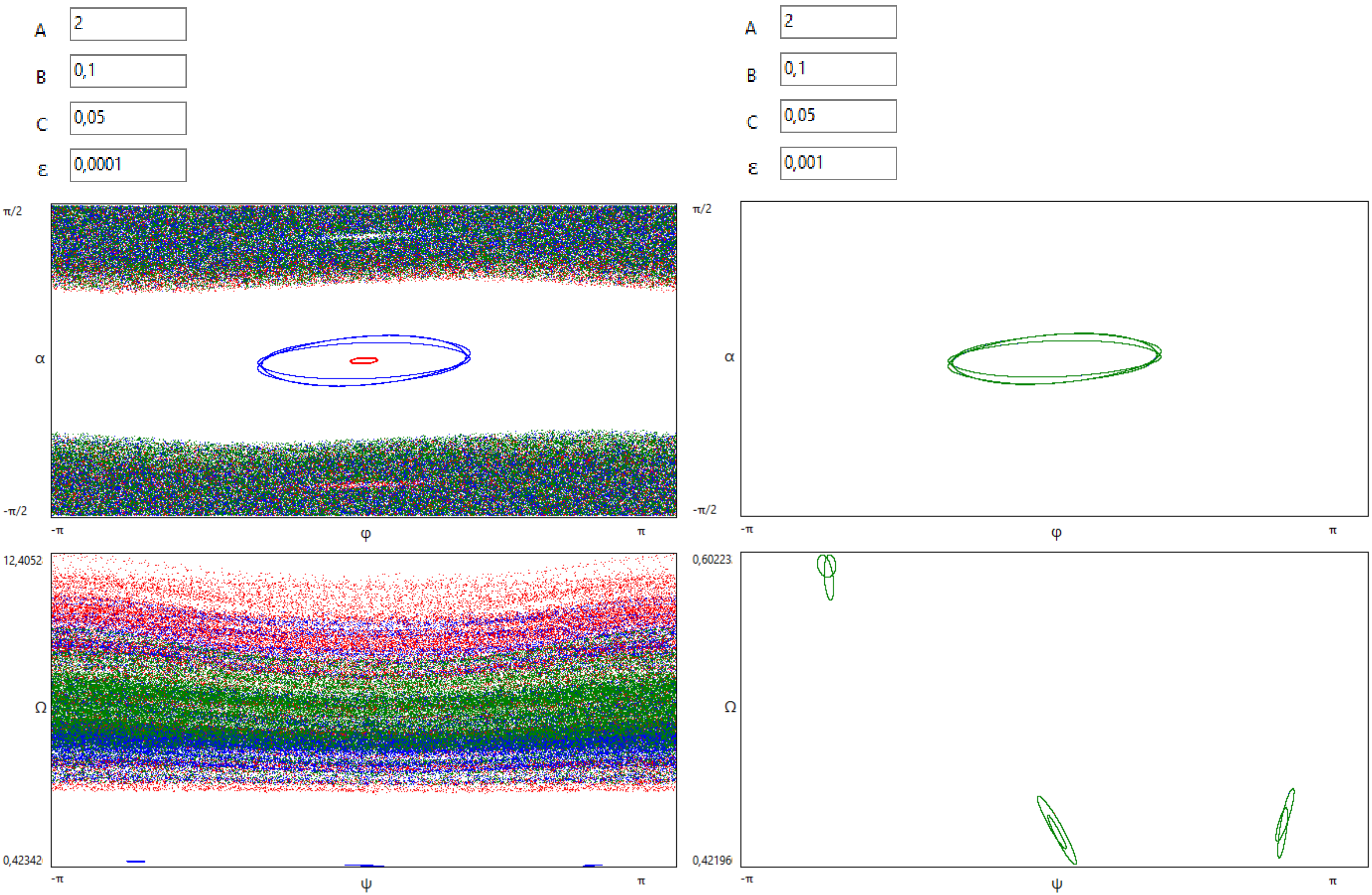
Blue	10
Green	50
Red	75
Purple	100
Yellow	61-67



Plot of dependence of the minimum in the ensemble, ensemble-averaged and maximum in the ensemble of the velocity after 2,000,000 iterations on the amplitude of the wall oscillation



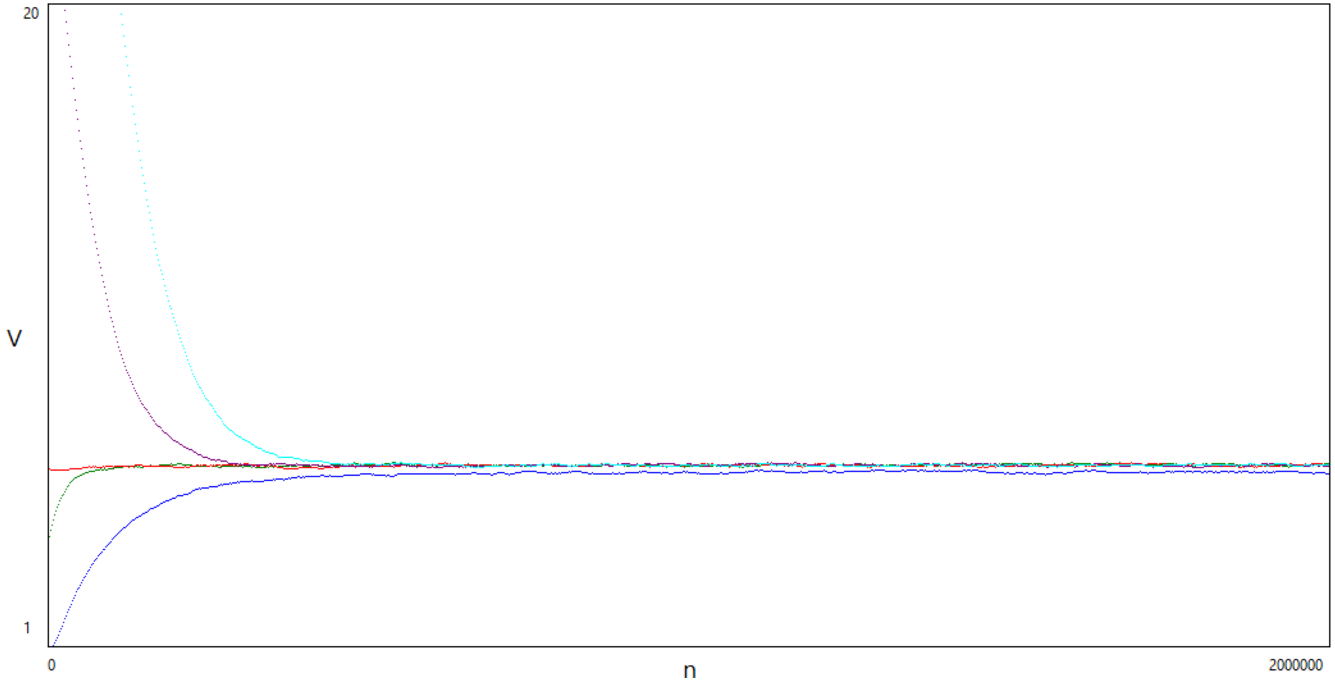
Attractor projections in a dissipative situation



Plots of dependence of the ensemble-averaged velocity on the number of iterations

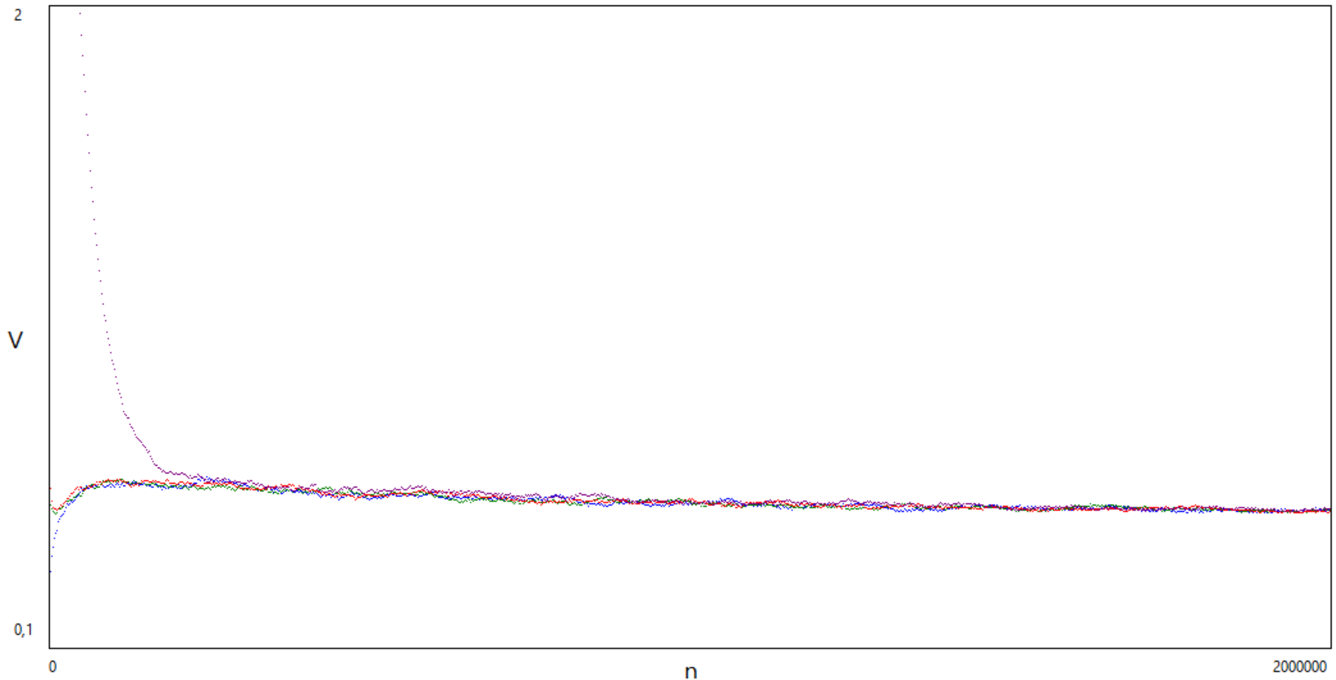
A: 2
B: 0,1
C: 0,05
ε: 0,0001

Blue	1.0
Green	4.0
Red	6.25
Purple	25
Light Blue	50

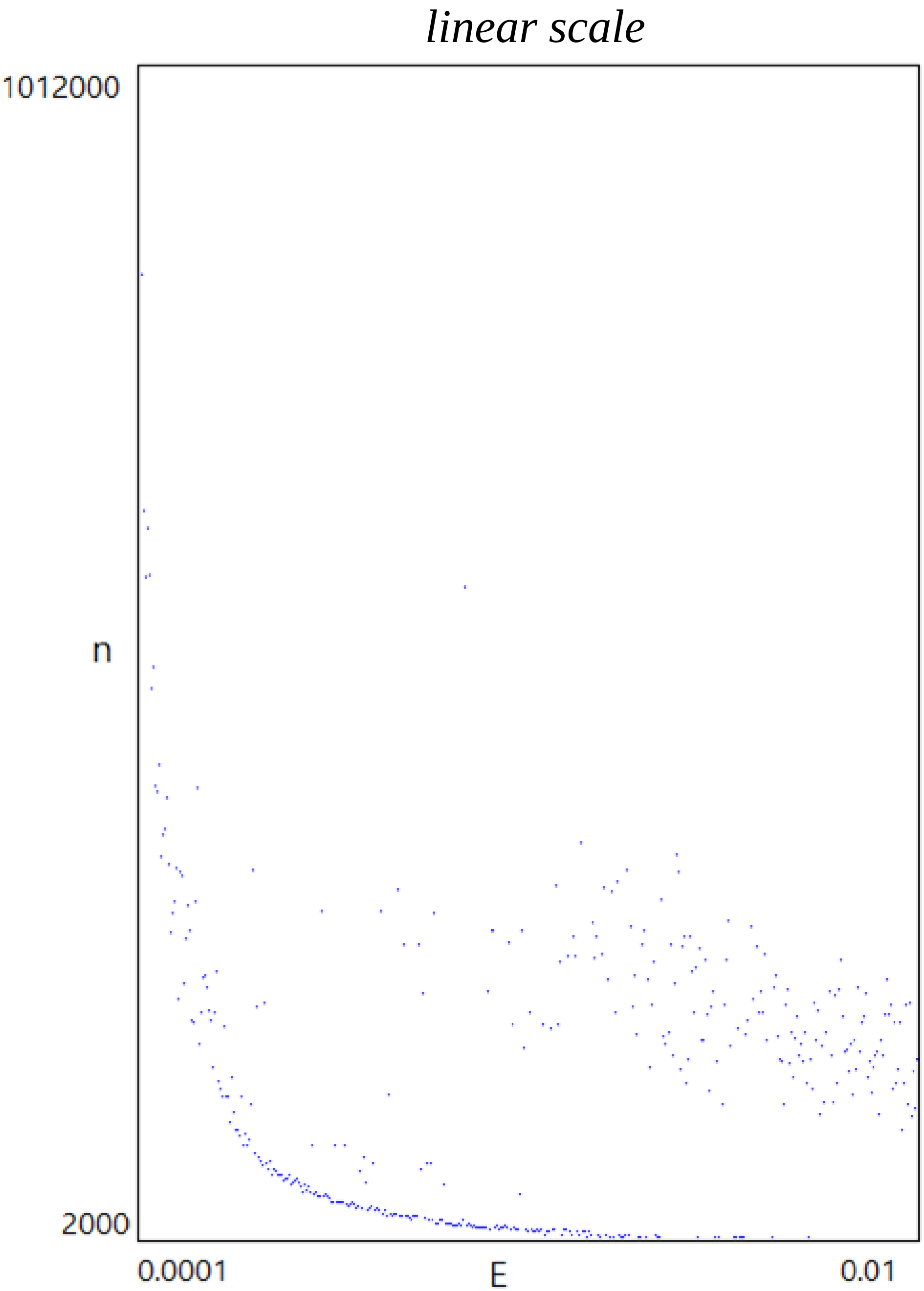


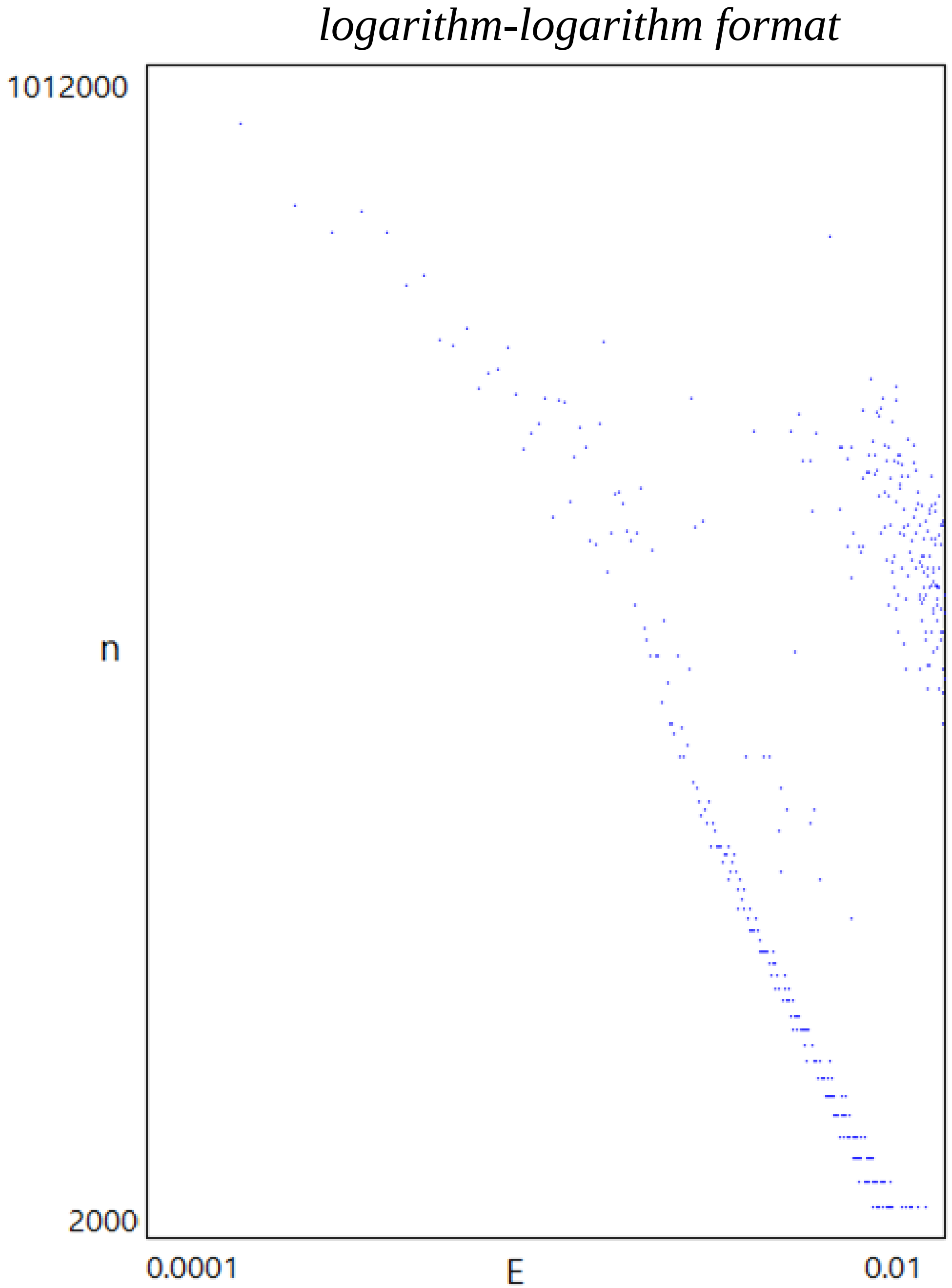
A: 2
B: 0,1
C: 0,05
ε: 0,001

Blue	10
Green	50
Red	75
Purple	100
Yellow	61-67



Plot of dependence of the stabilization time of the ensemble-averaged velocity on the dissipation parameter





Conclusions

1. Two-frequency tori are transformed into three-frequency tori when oscillations are introduced into the Tennyson - Lieberman - Lichtenberg model. Attractors, such as fixed points and limit two-frequency tori, are formed when dissipation is introduced.

2. The dependence of the ensemble-averaged and maximum in the ensemble of the velocities after 2,000,000 iterations on the amplitude of the wall oscillation is exponential in a large section.

3. The effect of "Billiard Maxwell demon" is observed at certain parameters. Basically, the system has limiting velocities to which all trajectories tend. These limiting velocities are distributed over a certain area.

4. The introduction of dissipation accelerates the stabilization of trajectories to limiting velocities. The dependence of the stabilization time on the dissipation parameter has a hyperbolic dependence over a significant section.