Extended version of detrended cross-correlation analysis

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***Abstract*—We describe an extended version of the detrended cross-correlation analysis which is useful for studying time-varying dynamics of complex systems producing inhomogeneous datasets. The method computes two independent measures that quantify the detrended covariance and the impact of nonstationarity, respectively. We apply this approach to characterize entrainment phenomena in the chaotic dynamics of two coupled Lorenz models with an additional trend in the datasets under study.**

***Index Terms*—cross-correlations, fluctuation analysis, scaling, nonstationarity**

I. Introduction

Cross-correlations typically appear in the behavior of coupled systems. Their characterization in terms of the classical cross-correlation function is restricted in the case of time-varying dynamics and nonstationary processes. Moreover, this function decreases for random processes, approaching zero with growing time delay, and the latter limits a reliable assessment of its features in this area. In particular, the characterization of long-range correlations becomes questionable. A way for providing a more authentic description of such correlations is based on the detrended fluctuation analysis (DFA) [1], [2], which is used in various research fields [3]–[5]. In addition to its original version, a modification was proposed for studying two nonstationary processes, called detrended cross-correlation analysis (DCCA) [6]. The given method is well adapted for the case of a rather homogeneous structure of the signals under study and comparable fluctuations from the local trend for different parts of data. This circumstance enables computing the averaged RMS fluctuations throughout the whole signals to estimate the global quantity (scaling exponent) describing long-range correlations within DFA (or cross-correlations for DCCA). If RMS fluctuations for some segments of the signal profile strongly outperform those for other parts of the data, then the estimation of the scaling exponents can result in misinterpretations of the correlation features. To take such circumstance into account, an extended DFA was proposed and applied to various types of signals [7], [8]. This approach introduces an additional scaling exponent which accounts for data inhomogeneity. For rather homogeneous data sets, the extended DFA does not enable obtaining new information and advantages over the standard DFA. But if there are clear inhomogeneities in the data sets under study, such new information will be associated with the impact of nonstationarity. We have also proposed a modified version for the case of two nonstationary signals, called extended detrended cross-correlation analysis (EDCCA) [9]. This modification computes two quantities associated with the detrended covariance and nonstationarity effects. Here, some features of EDCCA are described and illustrated using simulated datasets.

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II. Methods

Detrended fluctuation analysis is a way of quantifying long-range power-law correlations in nonstationary signals. This approach includes a construction of the profile of the signal $\{x_i\}$, i=$1,\ldots,N$

$$y_k = \sum_{i=1}^{k} x_i, \tag{1}$$

that is further divided into parts of equal length n. The local trend z_k is computed within each part due to the least-squares algorithm. Standard deviations of the profile from the local trend are used to determine the scaling exponent α [2]

$$F_{DFA}(n) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}[y_k - z_k]^2} \sim n^{\alpha}. \tag{2}$$

The extended version of DFA computes the difference between the maximum and minimum local standard deviation [7]

$$dF_{EDFA}(n) = \max\left[F_{loc}(n)\right] - \min\left[F_{loc}(n)\right]. \tag{3}$$

In order to analyze two time series $\{x_i\}$ and $\{\tilde{x}_i\}$, i=$1,\ldots,N$, both profiles are constructed

$$y_k = \sum_{i=1}^{k} x_i, \quad \tilde{y}_k = \sum_{i=1}^{k} \tilde{x}_i \tag{4}$$

and divided into segments. To increase their number, overlapping segments of $n-1$ values are considered [6] with computing the local linear trends z_k and $\tilde{z}_k$ within each part of the data. The detrended cross-correlation is computed for individual segments

$$f_{DCCA}^2(n,i) = \frac{1}{n-1}\sum_{k=i}^{i+n}(y_k - z_k)(\tilde{y}_k - \tilde{z}_k) \tag{5}$$

and the averaging procedure is carried out

$$F_{DCCA}^2(n) = \frac{1}{N-n}\sum_{i=1}^{N-n} f_{DCCA}^2(n,i). \tag{6}$$

The power-law dependence

$$F_{DCCA}(n) \sim n^{\lambda}. \tag{7}$$

enables computing the scaling exponent λ [6].

An extended DCCA [9] involves computations of differences between the extremal values of local standard deviations

$$\begin{aligned} dF(n) &= \max\left[F_{loc}(n)\right] - \min\left[F_{loc}(n)\right], \\ d\tilde{F}(n) &= \max\left[\tilde{F}_{loc}(n)\right] - \min\left[\tilde{F}_{loc}(n)\right], \end{aligned} \tag{8}$$

and estimating a measure

$$dF_{EDCCA}(n) = \sqrt{dF(n) * d\tilde{F}(n)} \sim n^{\mu}. \tag{9}$$

which often shows a power-law behavior quantified by the scaling exponent μ. A more stable approach deals with the standard deviations of F_{loc}

$$dF_{EDCCA}(n) = \sqrt{\sigma(F_{loc}(n)) * \sigma(\tilde{F}_{loc}(n))} \sim n^{\mu}, \tag{10}$$

since effects of artifacts will be reduced, if they still remain in datasets after the preliminary filtering procedures.

Datasets for cross-correlation analysis were selected from the dynamics of two interacted Lorenz models

$$\begin{aligned} \frac{dx_{1,2}}{dt} &= s(y_{1,2} - x_{1,2}) + \gamma(x_{2,1} - x_{1,2}), \\ \frac{dy_{1,2}}{dt} &= r_{1,2}x_{1,2} - x_{1,2}z_{1,2} - y_{1,2}, \\ \frac{dz_{1,2}}{dt} &= x_{1,2}y_{1,2} - z_{1,2}b, \end{aligned} \tag{11}$$

with the parameters s=10, r_1=28.8, r_2=28, b=8/3, and represented sequences of return times into the Poincaré sections $x_1^2 + y_1^2$=30 and $x_2^2 + y_2^2$=30. To account for nonstationarity, a trend was added to both sequences, which was selected as 1/4 period of the cosine function.

III. Results

Cross-correlation analysis of two sequences of return times associated with each Lorenz model enables separation between synchronous and asynchronous oscillations. The presence of trends makes the analysis more complicated because this nonstationarity affects estimations of the scaling exponents. However, such complication is mainly associated with the area of long-range correlations, where the estimated scaling exponents strongly outperform their expected values. When the range of scales decreases, effects of nonstationarity become weaker (e.g., for lg n<2.3), and distinctions between synchronous and asynchronous dynamics become well pronounced. In this range of scales, data analysis can be done without trend removal procedures. When larger scales should be considered, the preliminary filtering by a high-pass filter is required. The scaling exponent μ is more sensitive to this procedure according to its definition. Aiming to quantify effects of synchronization in terms of the scaling exponents, the region lg n<2.2 was chosen.

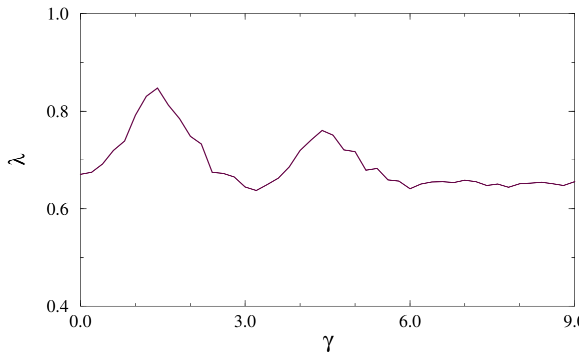

Fig. 1. Scaling exponent λ depending on the coupling strength

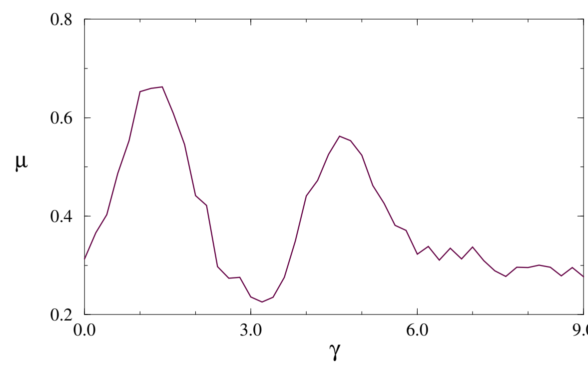

Fig. 2. Scaling exponent μ depending on the coupling strength

Figures 1 and 2 show changes in λ and μ caused by adjustment phenomenon when the coupling strength increases. Thus, for γ>6, both Lorenz models demonstrate synchronous chaotic dynamics, and further changes of γ do not affect the scaling exponents. The distinctions between asynchronous (e.g., γ=1.5) and synchronous (γ>6) oscillations are better quantified by μ-exponent which shows stronger changes due to the entrainment phenomenon. Note that these results are rather stable in the case of additive noise.

IV. Conclusion

Because nonstationarity can strongly affect the results of signal processing, approaches adapted for the analysis of time-varying dynamics become highly important in many areas of science, where real-world processes are analyzed. In this study we consider an approach for cross-correlation analysis which modifies the earlier proposed DCCA-method. This approach evaluates two scaling exponents, associated both with the original DCCA and impact of nonstationarity. The second exponent is non-informative for rather stationary processes and its evaluation is carried out only for inhomogeneous datasets. This approach was applied to quantify chaotic synchronization in the dynamics of two interacting Lorenz models with an additional nonstationarity. In particular, we have shown that the introduced scaling exponent μ is able to clearly quantify the entrainment phenomena, although it has a much wider field of possible applications in experimental studies.

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