The Impact of Interlayer Coupling Defects on Synchronisation in a Two-layer Network of Nonlocally Coupled Logistic Maps

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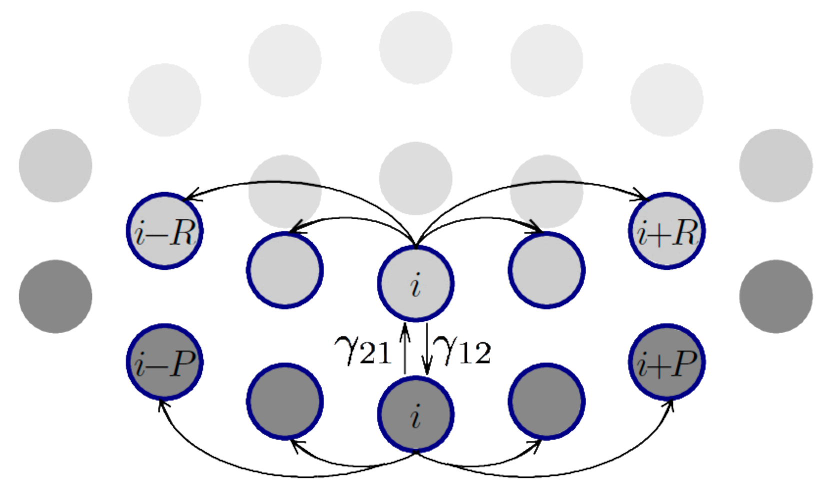
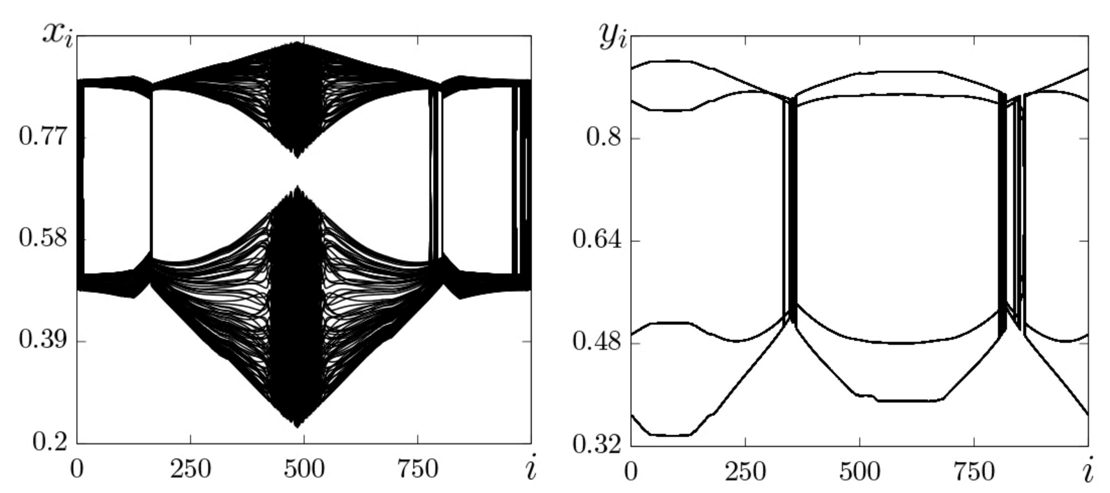
System under Study



$$x_i^{t+1} = f_i^t + \frac{\sigma_1}{2P}\sum_{j=i-P}^{i+P}\left[f_j^t - f_i^t\right] + \gamma_{21}F_i^t,$$

$$y_i^{t+1} = g_i^t + \frac{\sigma_2}{2R}\sum_{j=i-R}^{i+R}\left[g_j^t - g_i^t\right] + \gamma_{12}G_i^t,$$

We consider external and mutual synchronisation of chimera states in a two-layer network of oscillators for both unidirectional and bidirectional coupling. Individual elements represent dissipatively and nonlocally coupled chaotic oscillators.
In the system, a_1, a_2 – control parameters; σ_1, σ_2 – coupling strength between invidual elements; P, R – the number of neighbours to the left and right from the i-th element; γ_{12}, γ_{21} – intercoupling strength.
$F_i = (g_i - f_i)$, $G_i = (f_i - g_i)$ – coupling functions.
$f(x_i) = a_1 x_i(1-x_i)$, $g(y_i) = a_2 y_i(1-y_i)$ – maps defining individual elements.



Spatio-temporal profiles of the initial state of the layers. The parameters are:
N = 1000,
P = R = 320,
$a_1 = a_2 = 3.8$,
$\sigma_1 = \sigma_2 = 0.32$,
$\gamma_{12} = \gamma_{21} = 0$.

Defects Introduction

Phenomena that we observe in reality may vary from the ones that we study in a numerical experiment due to various reasons, e. g. energy dissipation, equipment damage, etc. In order to make the numerical experiment more realistic, it is crucial to examine the system in the presence of defects so it would be possible to predict the system's behaviour in situations where some connections are lacking.
We consider several cases of intercoupling defects:

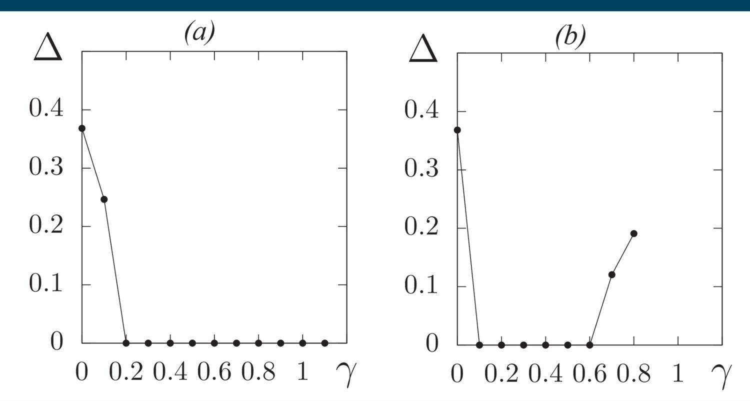
- Randomly distributed defects. We remove 25% and 50% of interconnections randomly.
- Evenly distributed defects. We remove 25% and 50% of interconnections evenly, i. e. every fourth and every second interconnection respectively.
- Cluster intercoupling. We divided the x_i ring into several clusters according to the type of structure that is being observed in its initial condition:
  - amplitude chimera state in the cluster from the 450th to the 520th particle,
  - phase chimera states in [0;10], [160; 170], [750; 810] and [960; 999],
  - coherent clusters in [10; 160] and [810; 960],
  - and incoherent clusters in [170; 450] and [520; 750].

  In this case the layers are connected only through the desired type of the cluster.

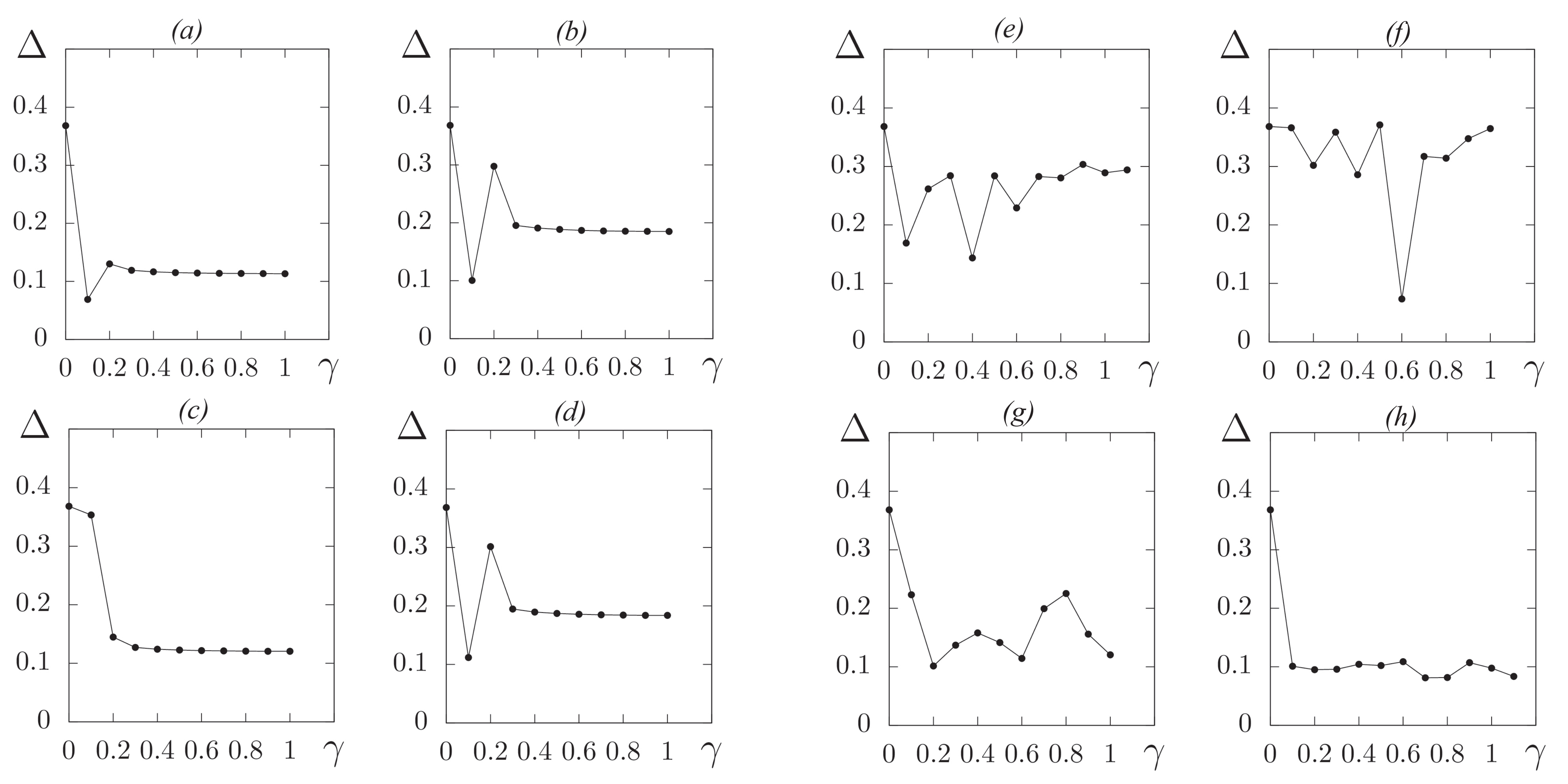
Synchronisation Estimation

The synchronisation effect is estimated by computation of sample standard deviation. (a) - unidirectional, (b) - bidirectional coupling with no defects.

$$\Delta = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{1}{n}\sum_{k=1}^{n}(y_i(k) - x_i(k))^2},$$



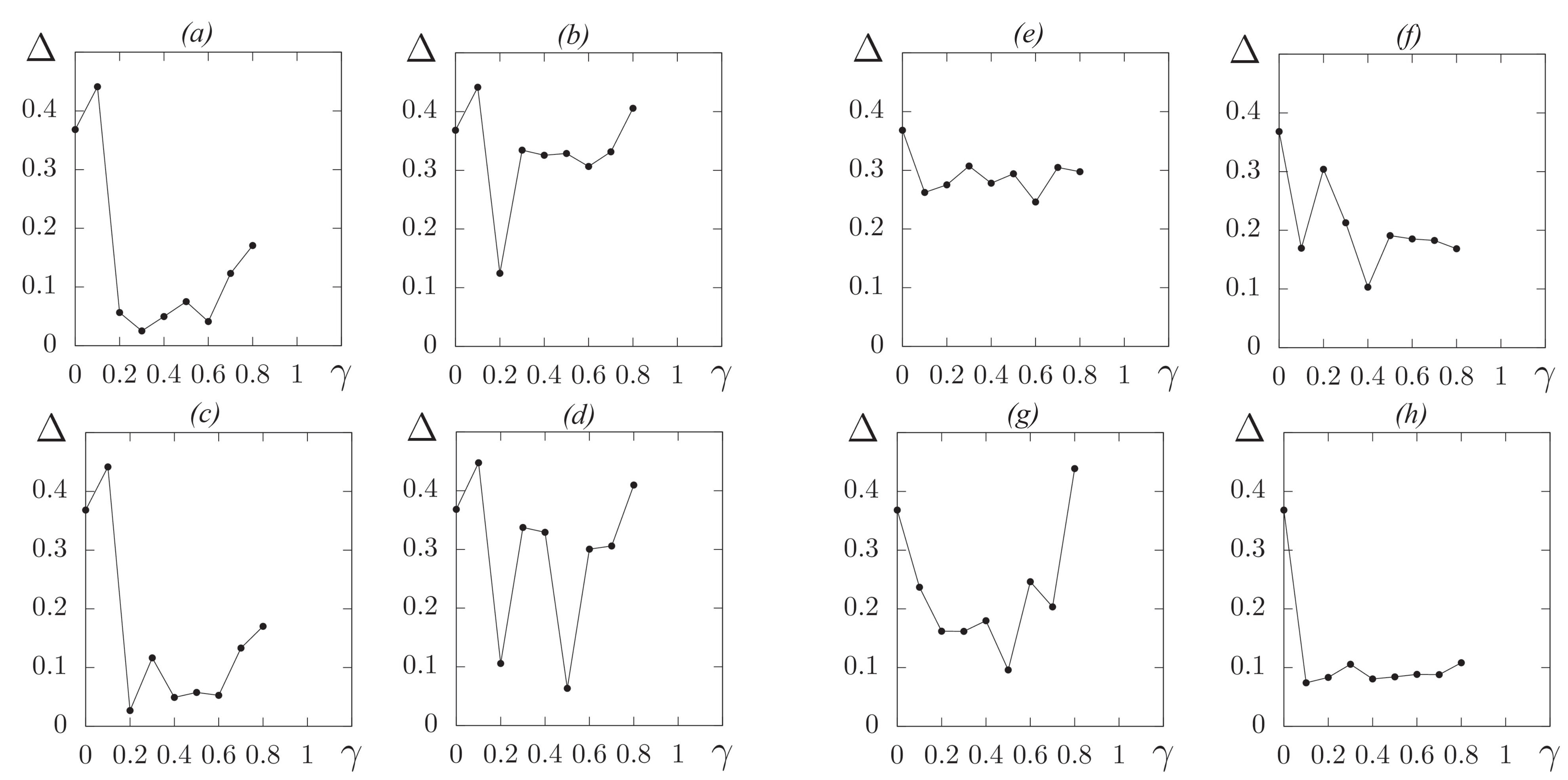
External Synchronisation



The results are represented by sample standard deviation △ graphs. When the intecoupling defects are distributed randomly (a - 25%, b - 50%) or evenly (c - 25%, d - 50%), one can notice that the average deviation value grows with the increase of the defects percentage. I. e. the more interconnections are lacking the worse the synchronisation effect is, as was expected.
On the next step we explored the so-called cluster intercoupling when the two ensembles were connected only via the amplitude chimera state (e), the phase chimera states (f), coherent (g) and incoherent (h) clusters. In this case the results are much more peculiar than in the previous ones.
When the layers are connected via the amplitude chimera state, it is possible to achieve structures similar to the initial state of the x_i ring in the y_i ring though these states cannot be called synchronous due to high values of △. The similar effect is observed in the case of coupling via the phase chimera states.
In the case of connection via coherent clusters the values of deviation are much smaller than previously, but the structures in the spatio-temporal profiles differ a lot from the initial structures of the x_i ring.
The intercoupling via incoherent clusters is most successful as the △ values are low and the structures in the y_i ring resemble the initial state of the x_i ring.

Mutual Synchronisation

In the case of mutual synchronisation with random (a - 25%, b - 50%) and even (c - 25%, d - 50%) defects distribution, with a general tendency for the deviation to grow with an increase in the number of defects, one can see anomalies in the form of areas with small deviation when defect percentage is significant.
The next step is cluster intercoupling when the two ensembles were connected only via the amplitude chimera state (e), the phase chimera states (f), coherent (g) and incoherent (h) clusters. The results are similar to those of external synchronisation with several differences.
The main difference lies in the structures that can be observed in spatio-temporal profiles of the layers. For instance, when the layers are connected via the amplitude chimera state and the intercoupling is unidirectional, the chimera state projects its chaotic regime onto the entirety of the y_i ring, while in the case of bidirectional interaction the complexity of this cluster varies and corresponds to the complexity of the whole layer's structure. The same effect is observed in the case (f).
In the case (g) the results are similar, however, there is an exception of γ value when there is an effective synchronous state with precision of △ = 0.09.
Similarly, in (h) there is an effective synchronous state with precision of △ = 0.08.



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