Dynamics of manylevel atoms in nonideal cavities

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ABSTRACT

A computer simulation of a system of two identical multilevel atoms in a nonideal resonator has been performed. Using the coherent states approach, the influence of atomic motion, cavity decay constants, and various initial field states on atomic entanglement and the chaos of atomic populations is investigated.

Keywords: two- and three-level atoms, atoms movement, non-ideal cavities, coherent states, entanglement, quantum chaos

1. INTRODUCTION

The interaction of atoms with photons is the main object of research in cavity quantum electrodynamics in the last thirty years. A very helpful model of this interaction was proposed more than fifty years ago by Jaynes and Cummings.[1] This model describing of two-level atom interacting with the photon mode in an ideal cavity plays the central role in modern quantum optics and laser physics. It predicts many fundamental effects such as Rabi oscillations in presence of vacuum cavity field, revivals and collapses of Rabi oscillations of atomic populations in presence of a cavity coherent field. These theoretical predictions were tested successfully in experiments with one-atom maser. The comprehensive overview of the JCM and some of the many extensions and generalizations may be found in the article.[2] The two-level model is physically realistic and important in modern quantum information schemes. It is well known that the basic element in quantum information science is a qubit (a two-level system).[3] The typical example of qubit in quantum optics is a two-level atom in an cavity.

Recently, see, for example, Refs.,[4–6] were considered the atom movement through the cavity and their influence on entanglement, discard and chaos in atomic population dynamics. In this work we consider the cases of two 2- and 3-level atoms, interacting with quantized electromagnetic fields of a cavities and their motions in classical approximation.

In our previous papers[7,8] the computer simulations of the Dicke model dynamics[9] and model of two identical two-level atoms in non-ideal cavity were performed. Influence of the distance between atoms, different cavity damping constant and different initial field mode state on entanglement were investigated using coherent states approach to quantum dynamics. The influence of the dynamical chaos of the classical analogue of considered quantum optical models (Dicke model and model Tavis - Cummings) on the entanglement of atomic and atomic-field states depending on the model parameters were simulated.

Group-theoretical coherent states (CS) have a very wide application in modern quantum optics and quantum information theory. It is well known, CS are the quantum states most close for classical ones. Because of this, CS basis is supposed to be quite relevant for quantum chaos researching. The approach we will follow here, consist in the study of semiclassical dynamics coherent states parameters of quantum systems that may be used in simulation of the dynamics of the relevant physical quantities. The coherent states are physically realized as a specific superpositions of the basic energy eigenvalue states, and they have close connections with unitary representations of dynamical groups for many important problems.[10–12]

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This paper presents the study of coherent states dynamics of three-level atoms in nonideal cavity. The interaction between atoms and cavity electromagnetic fields will be considered without the rotating wave approximation. We have derived and explored the dynamical (semiclassical) equations for the parameters of the CS of our system. Generally, there are a different ways to define the notion of chaos for quantum systems.[7,12]

The paper is organized as follows. Section 2 contains a brief mathematical introduction of group - theoretical coherent states connected with Heisenberg - Weyl group W_1 for photons in a cavity, for group $SU(2)$ - for two level atoms and group $SU(3)$ for three-level atoms. In the last case we shall deal with the "dressed" three-level analogy of Tavis - Cummings model (TCM)(out of "rotating-wave approximation"). In Section 3 a model Hamiltonian is presented, and we taken into account the movement of atoms through the cavity. In Section 4 we have have presented some results of our computer simulation of time dependencies for physical values important to description the dynamics of photons and atoms and maximal Lyapunov exponent calculation.

2. OSCILLATOR, TWO- AND THREE-LEVEL ATOMS COHERENT STATES

It has been noted in the introduction, the most natural way to study dynamics in quantum systems consists in using a group-theoretical coherent states technique.[11,12] If the Hamiltonian of a quantum system possesses dynamical symmetry group, this approach would be successfully applied.

For our purposes, we need three types of coherent states - oscillator (Glauber) - group W_1, spin -group $SU(2)$, and coherent states for three-level systems - group $SU(3)$.

In the case of two-level atom, interacting with photons in a cavity we need a coherent states in the form of a direct product of a oscillator CS $|\alpha>$ and spin CS $|z>$: $\quad |\alpha>\otimes|z>$. Here

$$|\alpha>=\exp\left(-|\alpha|^2/2\right)\cdot\exp\left(\alpha\hat{a}^{+}\right)|0>,\quad \hat{a}|0>=0,\quad <0|0>=1,\quad [\hat{a},\hat{a}_{+}]=\ 1, \tag{1}$$

$$|z>=(1-z\bar{z})^{-j}\cdot\exp\left(z\hat{J}_{+}\right)|J,-J>. \tag{2}$$

In the last equation $\hat{J}_{-}|J,-J>=0,\quad <J,-J|J,-J>=1,\quad [\hat{J}_{+},\hat{J}_{-}]=2\hat{J}_{3}$; $J=0,\ 1/2,\ 1,\ 3/2\ldots$.

The oscillator and spin CS are well known and have wide applications in modern quantum physics. Therefore, let us introduce the CS, applicable for the description of a 3-level systems.

Coherent states for a full symmetric irreducible representation $D(\mu,0)$ of $SU(3)$ are determined [12] by points of the quotient space $SU(3)/U(2)\ \approx CP_2$, local isomorphic to the two-dimensional complex space. The point of this space with coordinates $(z_1,\ z_2)$ corresponds to the vector of the CS

$$|z_1,z_2>=(1+z_1\bar{z}_1+z_2\bar{z}_2)^{-\mu/2}\ e^{z_2\hat{L}_{+}}\ e^{z_1\hat{J}_{+}}\ |0>. \tag{3}$$

For full symmetric irreducible representation $D(\mu,0)$, $\mu=2\mu_1-3\mu_2$, were μ_1 and μ_2 are the eigenvalues of the diagonal operators of the algebra $SU(3)$ defined in terms of the Cartan-Weyl realization ($\hat{H}_1|0\rangle=\mu_1|0\rangle$ and $\hat{H}_2|0\rangle=\mu_2|0\rangle$.), $\hat{L}_{+}$ and $\hat{J}_{+}$ raising operators of this algebra, which are not included in the maximum stationary subalgebra $U(2)$ of reference vector $|0>\equiv|E_0>$. It is chosen here as the vector corresponding to the lower level for the case of single atom (representation $D(1,0)$.

The set of CS possesses well-known properties (the resolution of unity) which is most important for our purpose.

$$\widehat{I}=\int\limits_{G/G_0} d\,\mu(CS)\,|CS><CS|,$$

where $d\mu(CS)$ is the invariant measure in the resolution of unity. In considered case of $G=W(2)\otimes SU(3)$

$$d\,\mu(CS)=\frac{d^2\alpha_1 d^2\alpha_2}{\pi^2}\,\frac{(\mu+2)(\mu+1)}{\pi^2}\,\frac{d^2z_1\,d^2z_2}{(1+z_1\bar{z}_1+z_2\bar{z}_2)^3},\quad \text{here}\quad d^2\xi\equiv d\mathrm{Re}(\xi)d\mathrm{Im}(\xi).$$

A group-theoretical coherent states matrix element of the system evolution operator

$\widehat{U}(t,t_0)=\exp[-i(t-t_0)\widehat{H}]$ is represented as

$$< CS|\exp[-i(t-t_0)\widehat{H}]|CS'>=\underbrace{\int\cdots\int}_{(N-1)} \quad < CS_N|\exp[-i\,(t-t_0)\,\widehat{H}/N]|CS_{N-1}> d\mu(CS_{N-1})\times$$

$$\times < CS_{N-1}|\exp[-i(t-t_0)\widehat{H}/N]|CS_{N-2}>\, d\mu(CS_{N-2})\times\ldots \tag{4}$$

$$\ldots\times d\mu(CS_1) < CS_1|\exp[-i(t-t_0)\widehat{H}/N]|CS_0>,$$

where $\Delta t=\frac{t-t_0}{N}, \quad N\gg 1.$

Then, we transform each factor in Eqs.(4)

$$< CS_{j+1}\,|\exp[-i(t-t_0\widehat{H}/N]|CS_j>\simeq< CS_{j+1}\,|\hat{1}-i(t-t_0)\widehat{H}/N\,|CS_j>\simeq$$

$$\simeq< CS_{j+1}|CSj>\,\exp[-i(t-t_0)\,\mathcal{H}(CS_{j+1}|CS_j)],$$

where

$$\mathcal{H}(CS_{j+1}|CS_j)\equiv\frac{< CS_{j+1}|\widehat{H}|CS_j>}{< CS_{j+1}|CS_j>}$$

is a symbol of Hamiltonian (matrix element of the Hamiltonian between CS).

In the limit $N\to\infty$ we get:

$$< \alpha,\zeta|\exp[-i(t-t_0)\widehat{H}]|\alpha^{'},\zeta^{'}>=\int DM_\infty\, e^{i\mathcal{S}},$$

where $\mathcal{S}$ - is treated as "classical" action function.

Let us consider a system of three-level atoms (qutrits) with energy levels $E_0<E_1<E_2$, placed in the model cavity having two resonant frequencies close to the frequencies of the two allowed atomic transitions. The cavity, we assume as nonideal. We will also take into account the non-resonant transitions. This model corresponds to the following generalization of the well-known model of Tavis-Cummings (see, for example, the paper[13]):

$$\left\{\begin{array}{l}\hat{H}=\hat{H}_0+\hat{V},\\ \hat{H}_0=\omega_0\hat{H}_1+\Omega_0\hat{H}_2+\omega_1\hat{a}_1^{+}\hat{a}_1+\omega_2\hat{a}_2^{+}\hat{a}_2,\\ \hat{V}=g_1(\hat{a}_1^{+}\hat{J}_-+\hat{a}_1\hat{J}_+)+f_1(\hat{a}_1^{+}\hat{J}_++\hat{a}_1\hat{J}_-)+g_2(\hat{a}_2^{+}\hat{L}_-+\hat{a}_2\hat{L}_+)+f_2(\hat{a}_2^{+}\hat{L}_++\hat{a}_2\hat{L}_-).\end{array}\right. \tag{5}$$

Here $\hbar=1$ and the Hamiltonian $\hat{H}_0$ describes free three-level atoms (first two terms) and two modes of the electromagnetic field in the cavity (last two terms), respectively. The operator $\hat{V}$ is determined by their interaction. For definiteness, we consider the transitions V - type - allowed transitions: $|E_0>\to|E_1>$ (connected with generators $\hat{J}_\pm$) and $|E_0>\to|E_2>$ (generators $\hat{L}_\pm$)) . Frequencies ω_0, Ω_0 are determined by the position of the atomic levels, $\omega_{1,2}$ - field frequencies, $g_{1,2}$ and $f_{1,2}$ are the constants of the "atom-field" interaction. The operators $\hat{H}_{1,2}$ are the diagonal generators of the group $SU(3)$ representation, describe the positions of the atomic levels. The operators $\hat{J}_\pm,\hat{L}_\pm$ are the generators of $SU(3)$ describing the transitions between the lower and the intermediate and lower and upper levels, respectively.

The constants $f_i=0$ in the resonant approximation and $f_i=g_i,\ (i=1,2)$, when the non-resonant transitions are taken into account.

The non-resonant terms are treated as corresponding to high-frequency processes, those contribution discards for the macroscopic time. But it is not correct, when coupling constant achieves value of order of atomic (or field) frequency. Here, the non-resonant terms' contribution is required taken into account. Recent experiments get hope to receive such (large) values of coupling constant.

To describe the dynamics of a three-level atoms, a system of coherent states for the group $SU(3)$ were constructed. This case we will look in more detail.

The selection rules allowed only two transitions between energy levels of a three-level system of only. As the result one may use the direct product of two-mode Weyl group $W(2)$ and $SU(3)$ group as a dynamical group for the study of three-level dynamics in a cavity.

Let us introduce a direct product of corresponding coherent states as a CS of the system:

$$|CS>=|\alpha_1,\,\alpha_2>\otimes|z_1,\,z_2>,$$

here

$$|\alpha_1,\,\alpha_2>=|\alpha_1>\otimes|\alpha_2>,$$

and $|\alpha_1,\,\alpha_2>$ is the usual 2-mode Glauber CS for a photons in a cavity, and $|z_1,\,z_2>$ is "atomic" CS, connected with irreducible representation of $SU(3)$ group.

Resulting equations of evolution of CS parameters take the following form:

$$\begin{cases} i\dot{\alpha}_1=\omega_1\alpha_1+\mu(g_1z_1+f_1\bar{z}_1)(1+z_1\bar{z}_1+z_2\bar{z}_2)^{-1}, \\ i\dot{\alpha}_2=\omega_2\alpha_2+\mu(g_2z_2+f_2\bar{z}_2)(1+z_1\bar{z}_1+z_2\bar{z}_2)^{-1}, \\ i\dot{z}_1=(\omega_0/2+\Omega_0)z_1-[(g_1\bar{\alpha}_1+f_1\alpha_1)z_1z_2+(g_2\bar{\alpha}_2+f_2\alpha_2)(z_1^2-1)], \\ i\dot{z}_2=(\omega_0+\Omega_0)z_2-[(g_1\bar{\alpha}_1+f_1\alpha_1)(z_2^2-1)+(g_2\bar{\alpha}_2+f_2\alpha_2)z_1z_2]. \end{cases} \tag{6}$$

In the rotating wave approximation $f_{1,2}=0$, and in general non-resonant case $f_{1,2}=g_{1,2}$.

The Eqs.(6) can be reduced to standard Hamiltonian form well known in classical mechanics:

$$\dot{Z}_s=\{Z_s,\mathcal{H}\},\quad \dot{\bar{Z}}_s=\left\{\bar{Z}_s,\mathcal{H}\right\}, \tag{7}$$

here $Z=(\alpha_1,\alpha_2,z_1,z_2)$, and Hamiltonian function $\mathcal{H}$ is equal to the matrix element:

$$\mathcal{H}=<CS|\hat{H}|CS>,$$

and the Poisson brackets $\{\,,\}$ are defined as:

$$\{F_1(Z,\bar{Z}),\,F_2(Z,\bar{Z})\}=\frac{1}{i}\sum_{s,s'}g^{ss'}(Z,\bar{Z})\left(\frac{\partial F_1}{\partial Z_s}\frac{\partial F_2}{\partial\bar{Z}_{s'}}-\frac{\partial F_2}{\partial Z_s}\frac{\partial F_1}{\partial\bar{Z}_{s'}}\right), \tag{8}$$

where $g_{s\,s'}=\partial^2\,\log\,K(Z,\bar{Z})/\partial Z_s\partial\bar{Z}_{s'}$, matrix $(g^{s\,s'})$ is reverse to matrix $(g_{s\,s'})$, and

$$K(Z,\bar{Z})=|<\mathbf{0}|Z>|^{-2},\quad |Z>\equiv|CS>.$$

In our case

$$K(Z,\bar{Z})=\exp\left(\alpha_1\,\bar{\alpha}_1+\alpha_2\,\bar{\alpha}_2\right)\,(1+z_1\bar{z}_1+z_2\bar{z}_2)^{\mu}.$$

The function $F(Z,\bar{Z})=<CS|\widehat{F}|CS>$ is well known as $Q-$symbol of operator $\widehat{F}$.

It can be shown that any appropriate system of physical operators $\{\widehat{F}_1,\ldots\widehat{F}_M\}$ is consistent with semiclassical equations of motion deduced from Hamilton equations with the Poisson brackets (8).

Therefore, we may understood by quantum chaos the stochastic behavior (exponential instability) of the CS parameters as well as chaotic regimes of these semiclassical equations.

Derived equations (6) have not an analytical solutions and we should use numerical methods. Unfortunately, the interpretation of the solutions of equations (6) is more complicated in comparison with a two-level systems,

because the $SU(3)$ atomic CS now defines by a point in two-dimensional complex (four-dimensional real) space of the $SU(3)/U(2)$, and build a separate trajectories in the complex planes for variables z_1 and z_2 has no meaning. When $f_{1,2} = 0$ the system is regular. If $f_{1,2} = g_{1,2}$ has become a chaotic for large enough of the interaction constants $g_{1,2}$ the system. We have computed a phase portraits and Poincare sections of real and imaginary parts of atomic CS parameter components for different values μ, and coupling constants, together with temporal dependencies of level populations and mean number of photons in the both modes in a cavity. In this publication (see, the section 4), we restrict ourselves to display the result of computer computation of the maximal Lyapunov exponent for the system of two three-level atoms ($\mu = 2$). The calculation of the Lyapunov exponent is conceptually simple since one only needs to follow two initially nearby trajectories and fit the logarithm of their separation to a linear function of time. The slope of the fit is the Lyapunov exponent. The maximal Lyapunov exponent λ_{max} is the largest one. In nonlinear dynamics it is well known that the case $\lambda_{max} > 0$ is the signal of the chaotic behavior of system.

To calculation of λ_{max} let we have a n- dimensional dynamical system with continuous time. To characterize the stability of its solutions, we need to analyze the temporal evolution of the infinitesimal n- dimensional sphere of the initial conditions. Over time, this sphere is transformed into an ellipsoid. The Lyapunov exponent λ_i can be defined in terms of the long axes of the ellipsoid $L_i(t)$:

$$\lambda_i = \lim_{t\to\infty} \log\left(\frac{L_i(t)}{L_i(0)}\right),$$

Thus, the Lyapunov exponents are determined by the expansion or contraction of the scope in different directions in the phase space. In our case we have the eight-dimensional system of real and imaginary parts of the coherent stare parameters α_1, α_2, z_1, and z_2.

The calculated maximum Lyapunov exponent has shown that for large values of $f_1 = g_1, f_2 = g_2$ tends to unity, in the systems develops a pronounced chaos.

We presented here only simplest three-level system (see, Eqs. (5)), when all atoms are equivalent. In the next section we describe some generalizations including different places and motion of atoms and losses for the photons, and the same for the case of two-level atoms.

3. MODEL HAMILTONIANS FOR MOVING ATOMS

3.1 Two - Level Moving Atoms

The Hamiltonian for the two moving 2-level atoms interacting with a cavity photons is the following (here and below $\hbar = 1$):

$$\hat{H}(t) = \omega\hat{a}^{+}\hat{a} + \omega_0\left(\hat{J}_3^{(1)} + \hat{J}_3^{(2)}\right) + \left\{g_1(t)\left(\hat{J}_{+}^{(1)} + \hat{J}_{-}^{(1)}\right) + g_2(t)\left(\hat{J}_{+}^{(2)} + \hat{J}_{-}^{(2)}\right)\right\}\cdot\left(\hat{a}^{+} + \hat{a}\right). \qquad (9)$$

Here we have the dynamical group $G = W_1 \times SU(2) \times SU(2)$, and

$$[\hat{J}_{+}^{(\sigma)}, \hat{J}_{-}^{(\kappa)}] = 2\hat{J}_3^{(\sigma)}\cdot\delta_{\sigma\kappa},\ [\hat{J}_3^{(\sigma)}, \hat{J}_{\pm}^{(\kappa)}] = \pm\hat{J}_{\pm}^{(\sigma)}\cdot\delta_{\sigma\kappa} \quad \sigma,\kappa = 1,\ 2.$$

In the formula (9) we introduced the time dependencies of the atom-field interaction constants:

$$g_1(t) = g\cdot[\theta(t) - \theta(t - l/v_1]\cdot\sin\left(\frac{\pi v_1}{l}t\right), \quad g_1(t) = g\cdot\left[\theta(t - t_2^0) - \theta(t - t_2^0 - l/v_2\right]\cdot\sin\left(\frac{\pi v_1}{l}(t - t^0 \quad {}_2)\right), \qquad (10)$$

where l is the size of the cavity along the path of atoms, $v_{1,2}$ are the atom velocities, and t_2^0 is the appearance time of the second atom into the cavity, provided that the first atom in it falls at time $t = 0$. In Exp. (10) the function $\theta(\tau)$ is the well known Heaviside step function.

A system of equations for CS parameters $|Z>=|\alpha, z^{(1)}, z^{(2)}>\equiv|\alpha>\otimes|z^{(1)}>\otimes|z^{(2)}>$ (here $|z^{(\sigma)}>?\,\sigma =$ 1, 2 is the CS for a single 2-level atom? for which $J = 1/2$) have the next view:

$$\begin{cases} i\dot{\alpha} = \omega\alpha + g_1(t)\frac{z^{(1)}+\bar{z}^{(1)}}{1+z^{(1)}\bar{z}^{(1)}} + g_2(t)\frac{z^{(2)}+\bar{z}^{(2)}}{1+z^{(2)}\bar{z}^{(2)}}, \\ i\dot{z}^{(1)} = \omega_0 z^{(1)} + g_1(t)(\alpha+\bar{\alpha})\cdot(1-(z^{(1)})^2), \\ i\dot{z}^{(2)} = \omega_0 z^{(2)} + g_2(t)(\alpha+\bar{\alpha})\cdot(1-(z^{(2)})^2). \end{cases} \tag{11}$$

The Hamiltonian (9) and the system (11) are considered without the resonant approximation, which is usually used in the Jaynes - Cummings model. In order to take into account the loss of photons in the cavity, we use the approach of the Wigner - Weisskopf approximation with substitutions $\omega \to \omega - i\gamma$, $\gamma \ll \omega$, where γ is the damping constant.

3.2 Three-Level Atoms

The Hamiltonian of the two 3-level $V-$atoms moving in a cavity is chosen in a similar expression (5), but in absence of the resonance approximation, and, as in 2-level case, with a time dependent atom-field interaction coefficients:

$$\begin{aligned} \hat{H}(t) = \omega_1\hat{a}_1^+\hat{a}_1 + \omega_2\hat{a}_2^+\hat{a}_2 + \omega_0\left(\hat{H}_1^{(1)} + \hat{H}_1^{(2)}\right) + \Omega_0\left(\hat{H}_2^{(1)} + \hat{H}_2^{(2)}\right) + \\ + \left\{g_1^{(1)}(t)\left(\hat{J}_+^{(1)} + \hat{J}_-^{(1)}\right) + g_1^{(2)}(t)\left(\hat{J}_+^{(2)} + \hat{J}_-^{(2)}\right)\right\}(\hat{a}_1^+ + \hat{a}_1) + \\ + \left\{g_2^{(1)}(t)\left(\hat{L}_+^{(1)} + \hat{L}_-^{(1)}\right) + g_2^{(2)}(t)\left(\hat{L}_+^{(2)} + \hat{L}_-^{(2)}\right)\right\}(\hat{a}_2^+ + \hat{a}_2). \end{aligned} \tag{12}$$

In the formula (12) all parameters are the same as in (5). The superscripts (1) and (2) are connected to the first and second atom, respectively. The time dependent interaction constants are the following:

$$g_1^{(1)}(t) = g_1\cdot[\theta(t) - \theta(t-l/v_1]\cdot\sin\left(\frac{\pi v_1}{l}t\right),$$
$$g_1^{(2)}(t) = g_1\cdot\left[\theta(t-t_2^0) - \theta(t-t_2^0-l/v_2\right]\cdot\sin\left(\frac{\pi v_1}{l}(t-t^0{}_{2})\right),$$
$$g_2^{(1)}(t) = g_2\cdot\left[\theta(t) - \theta(t-l/v_1\right], \quad g_2^{(2)}(t) = g_2\cdot\left[\theta(t-t_2^0) - \theta(t-t_2^0-l/v_2\right].$$

Introducing the coherent states $|Z>=|\alpha_1>\times|\alpha_2>\times|z_1^{(1)}, z_2^{(1)}>\times|z_1^{(2)}, z_2^{(2)}>$ associated with the dynamical group of the problem $G = W_2\times SU(3)\times SU(3)$ (here $|z_1^{(\sigma)}, z_2^{(\sigma)}>$, $\sigma = 1, 2$ is the $SU(3)$ CS for a single 3-level atom (see, Eqs (3)) with parameter $\mu = 1$), we obtain a system of equations:

$$\begin{cases} i\dot{\alpha}_1 = \omega_1\alpha_1 + g_1^{(1)}(t)\frac{z_1^{(1)}+\bar{z}_1^{(1)}}{1+z_1^{(1)}\bar{z}_1^{(1)}+z_2^{(1)}\bar{z}_2^{(1)}} + g_1^{(2)}(t)\frac{z_1^{(2)}+\bar{z}_1^{(2)}}{1+z_1^{(2)}\bar{z}_1^{(2)}+z_2^{(2)}\bar{z}_2^{(2)}}, \\ i\dot{\alpha}_2 = \omega_2\alpha_2 + g_2^{(1)}(t)\frac{z_2^{(1)}+\bar{z}_2^{(1)}}{1+z_1^{(1)}\bar{z}_1^{(1)}+z_2^{(1)}\bar{z}_2^{(1)}} + g_2^{(2)}(t)\frac{z_2^{(2)}+\bar{z}_2^{(2)}}{1+z_1^{(2)}\bar{z}_1^{(2)}+z_2^{(2)}\bar{z}_2^{(2)}}, \\ i\dot{z}_1^{(1)} = \Omega_1 z_1^{(1)} - g_1^{(1)}(t)(\alpha_1+\bar{\alpha}_1)z_1^{(1)}z_2^{(1)} + g_2^{(1)}(t)(\alpha_2+\bar{\alpha}_2)\left(z_1^{(1)}z_1^{(1)}-1\right), \\ i\dot{z}_2^{(1)} = \Omega_2 z_2^{(1)} + g_1^{(1)}(t)(\alpha_1+\bar{\alpha}_1)\left(z_2^{(1)}z_2^{(1)}-1\right) - g_2^{(1)}(t)(\alpha_2+\bar{\alpha}_2)z_1^{(1)}z_2^{(1)}, \\ i\dot{z}_1^{(2)} = \Omega_1 z_2^{(1)} - g_1^{(2)}(t)(\alpha_1+\bar{\alpha}_1)z_1^{(2)}z_2^{(2)} + g_2^{(2)}(t)(\alpha_2+\bar{\alpha}_2)\left(z_1^{(2)}z_1^{(2)}-1\right), \\ i\dot{z}_2^{(2)} = \Omega_2 z_2^{(2)} + g_1^{(2)}(t)(\alpha_1+\bar{\alpha}_1)\left(z_1^{(2)}z_2^{(2)}-1\right) - g_2^{(2)}(t)(\alpha_2+\bar{\alpha}_2)z_1^{(2)}z_1^{(1)}. \end{cases} \tag{13}$$

Also, as for the system of two 2-level atoms, we use in calculation the Wigner - Weisskopf approximation for photon modes with substitutions: $\omega_1 \to \omega_1 - i\gamma_1$, $\omega_2 \to \omega_2 - i\gamma_2$.

4. RESULTS OF COMPUTER SIMULATIONS

Performed computer simulations led to the following results.

The calculations for all systems were carried out for sufficiently large and for small (of the order of unity) of the initial value of the average number of photons in the field modes. Semiclassical consideration in this case is less relevant, but it makes sense to analyze the behavior of the actual model. It turned out that the destruction of the harmonic behavior of the transition probability of excited atoms in the non-excited state occurs at the same order of the coupling constants values, but more clearly manifests the following features: after the transition from nearly sinusoidal regime to multifrequency there exist a values of the coupling constants for which the number of frequencies in multifrequency mode is sharply reduced and the dynamics of the atomic subsystem becomes more regular. If one continue to increase the coupling constants, then there is a return to the multifrequency mode. The calculations have shown that the behavior of CS for the three-level atoms and two-level atoms is largely the same. However, there is one significant difference. If one select a constants g significantly different for different transitions in 3-level atoms, then the inclusion of transition with small coupling constant, leads to some suppression of the chaotic regime at the transition with greater interaction constant. This effect can be explained by the fact that the competing transition with small coupling constant leads to a decrease of the effective transition probability between levels due to the terms proportional to greater interaction constant and there is an effect of regularization. If, the both coupling constants have the same order and large enough, then there is no regularization of the randomness of both transitions.

Due to the limited volume of the article, we present here only a few illustrations from their very large amount, with the temporary dynamics just for the case of two two-level atoms.

On Fig.1 - Fig.3 we show the results of the calculation of the oscillator and atoms CS trajectories, and the time dependencies of the average number of the cavity photons and the populations of the upper atomic levels with different atom-field interaction constant g and the atoms velocity (for the case $v_1 = v_2 = v$). We use in the calculation of the dimensionless parameters, setting $\hbar = \omega_0 = 1$. Although the interaction constant g is chosen large enough, the temporal behavior of the model is regular, if the velocity (more accurately the value of the parameter $\beta = v/l$) is large. On the contrary, if the parameter $beta = v/l$) is small enough, than for $g \sim 0.7$, the time dependence of the upper atom populations acquires a multi-frequency mode.

To prove that in the non-resonance case for a system of the 3-level atoms we have dynamical chaos for the large enough interaction constants $g_{1,2}$, it is necessary to calculate the maximum Lyapunov coefficient. The result of the one calculation we show on Fig.4, were it is shown a density plot of λ_{max} on the plane of the interaction constants (g_1, g_2) for two 3-level atoms moved thru the cavity with small velocity.

For the values of these parameters between 0 and about 0.5 regular behavior alternate with irregular one, which corresponds to the well-known in the theory of chaotic systems phenomenon of intermittency. This correlates with the observed in our calculations of the time behavior of the population of the atoms on the upper level. It should be noted that the nature of intermittency, we are not yet quite clear for us, and to clarify the situation, more research is needed.

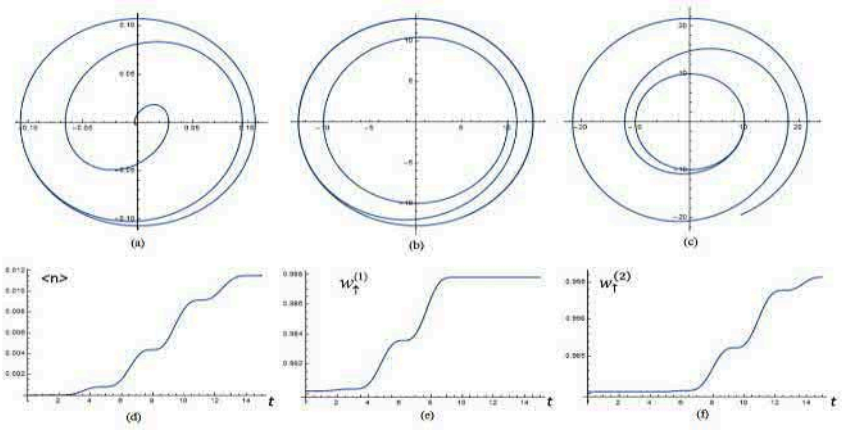

Figure 1: CS trajectories, time dependence of the mean number of photons and populations of the upper atom levels. $(a)-$ trajectory of CS on α- plane; $(b)-$ trajectory on z_1- plane; $(c)-$ trajectory on z_2- plane; $(d)-$ time dependence of mean number of photons; (e) and (f)- the plots of the upper level populations for the first and the second atom, respectively. Here $\alpha(0) = 0$, $z_1(0) = z_2(0) = 10$, $\omega_0 = 1$, $\omega = 0.999$, $\gamma = 0.001$, $g = 0.2$, $\beta = v/l = 0.1$

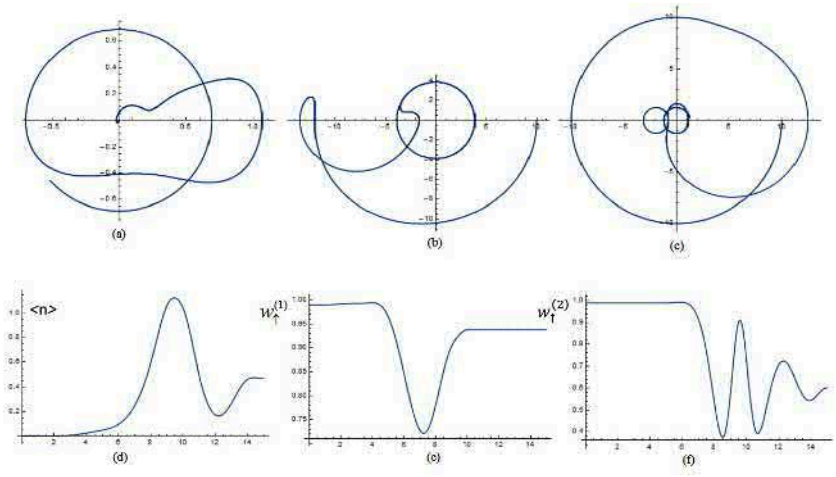

Figure 2: Same as in the Figure 1, with parameters $\alpha(0) = 0$, $z_1(0) = z_2(0) = 10$, $\omega_0 = 1$, $\omega = 0.999$, $\gamma = 0.001$, $g = 0.7$, $\beta = 0.1$

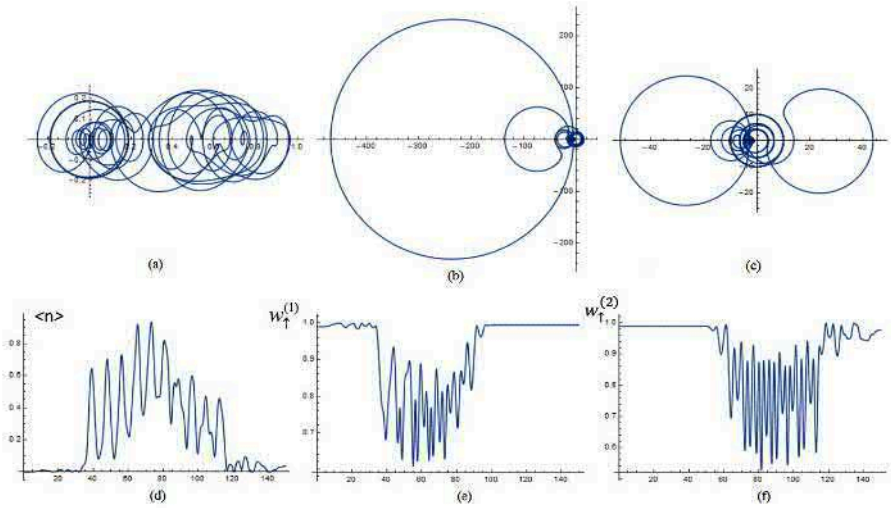

Figure 3: Same as in Figure 1, with parameters $\alpha(0) = 0$, $z_1(0) = z_2(0) = 10$, $\omega_0 = 1$, $\omega = 0.999$, $\gamma = 0.001$, $g = 0.7$, $\beta = 0.01$

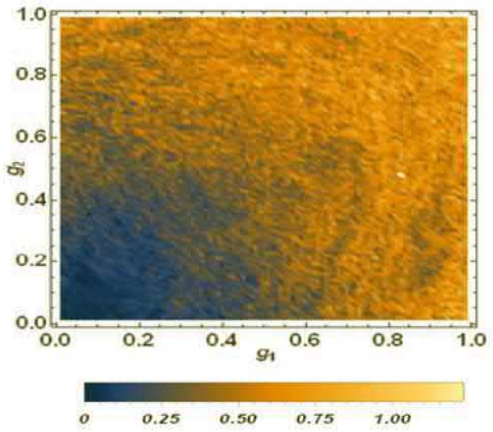

Figure 4: The density plot of maximal Lyapunov exponent λ_{max} on plane of interaction constants (g_1, g_2) for model with two 3-level atoms. $\omega_0 = 0.6$, $\Omega_0 = 0.4$, $\omega_1 = 1$, $\omega_2 = 0.7$. The legend show the interval of variation of λ_{max}. The parameter $\mu = 2$

CONCLUSIONS AND OUTLOOK

In this paper were obtained a new nonlinear semiclassical equations for two- and three-level atomsinteracting with photons in a cavities in the cases of motion of atoms. But these models not include direct interaction between atoms. Account of the dipole - dipole interaction and consideration of the influence of chaotic behavior of three-level systems in non-regular case on entanglement are a problem for our future investigation. Also relevant is the problem of calculating the influence of chaos on the entanglement in three-level atom systems with losses and memory effects (non-Markovian relaxation). It is also very interesting to study the effect of the initial correlations of atoms and photons on the properties of the quantum chaos.

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