

**Abstract** Nowadays, the problem of forecasting complex signals is significant and has many applications in real life. One of such applications is the prediction of neurophysiological signals, like EEG. Such signals are macroscopic signals of a group of neurons, and the connections between them adapt in time. Here, we investigate the possibility of forecasting the dynamics of the modulated adaptive network, which topology changes in time, using Reservoir Computing (RC). We show that the dynamics of the signal is chaotic, and RC cannot predict it, but reconstruction of the phase space by adding the delays improves the quality of the signal's prediction.

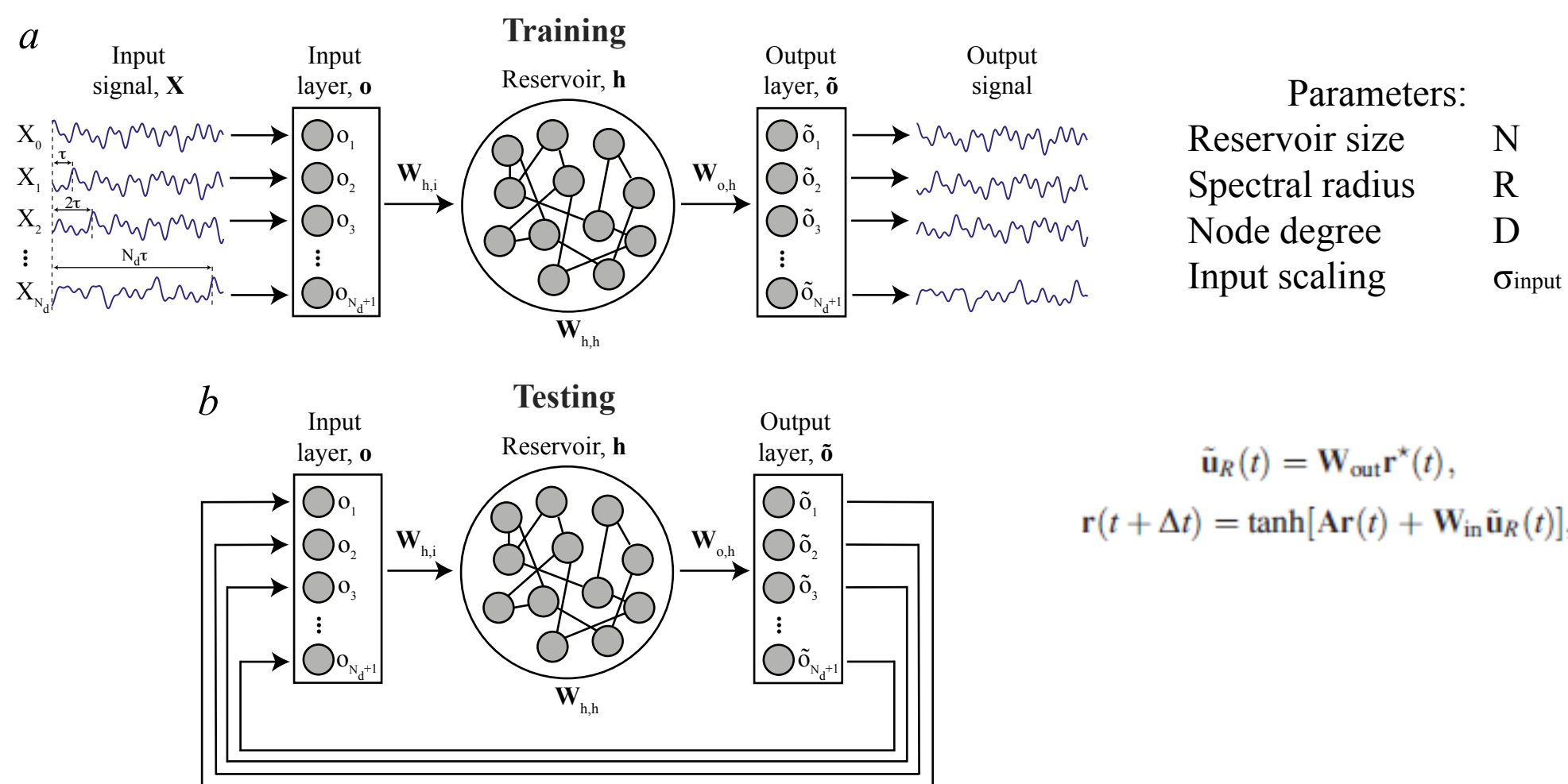


Fig. 1. (a) Schematic presentation of the RC network in the training mode and (b) predicting (testing) mode. The number of input signals is equal to  $(N_d + 1)$ , where  $N_d$  is the number of the delay coordinates.

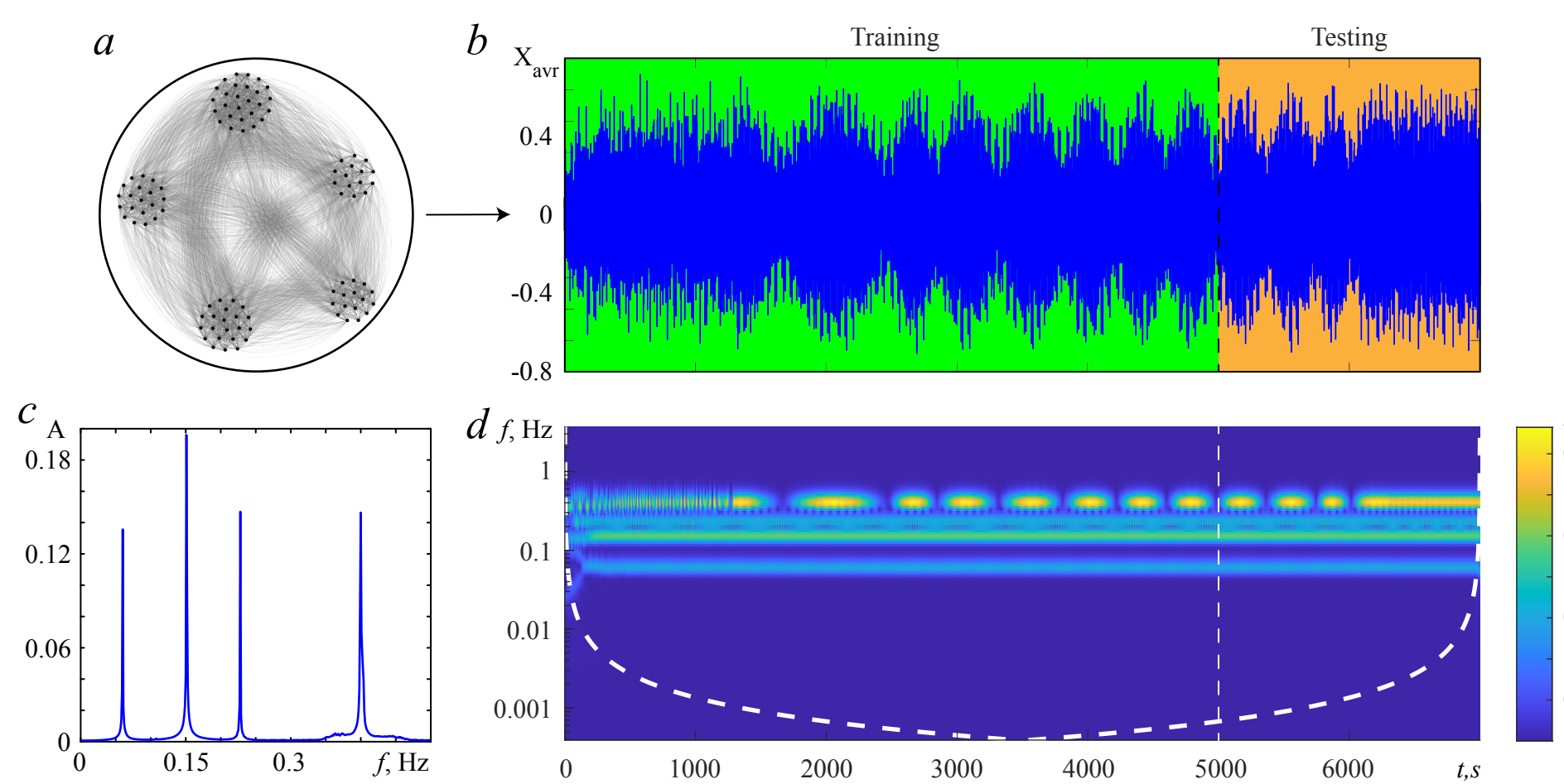


Fig. 2. (a) Schematic presentation of the topology of 100 Kuramoto phase oscillators network with the adaptation of the couplings. (b) The macroscopic signal obtained from the Kuramoto network by averaging the dynamics of all network's elements. One part of the signal is used in the training process, another one - in the predicting process. (c) Fourier and (d) wavelet spectra of the considered macroscopic signal.

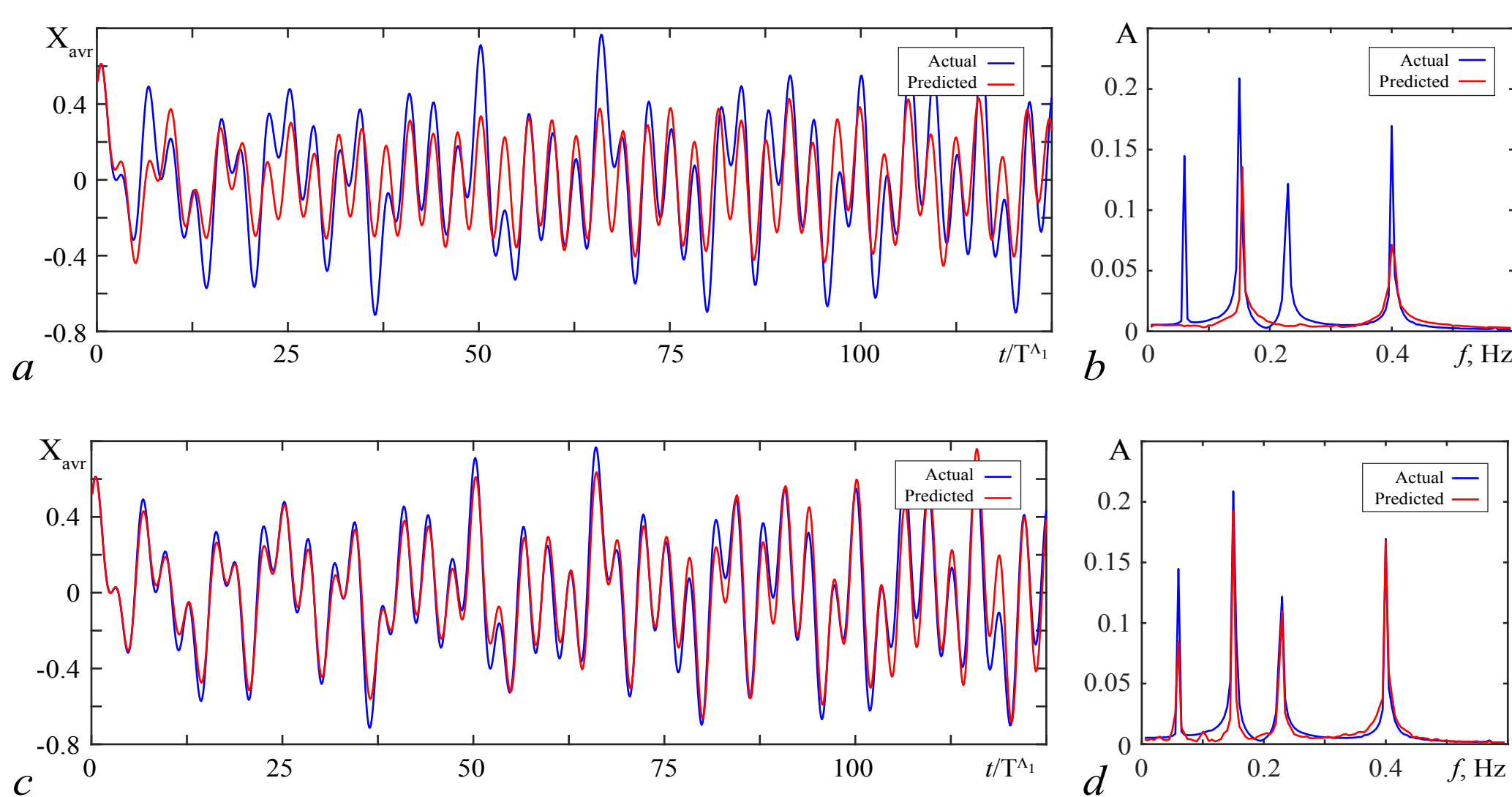


Fig. 3. (a,c) The state prediction (red) of the reservoir and the actual trajectory (black) of the Kuramoto network and (b,d) the corresponding amplitude spectra for 2 cases: (a,b) when we use only 1 original signal as the input one and (c,d) when add 2 delayed signals.

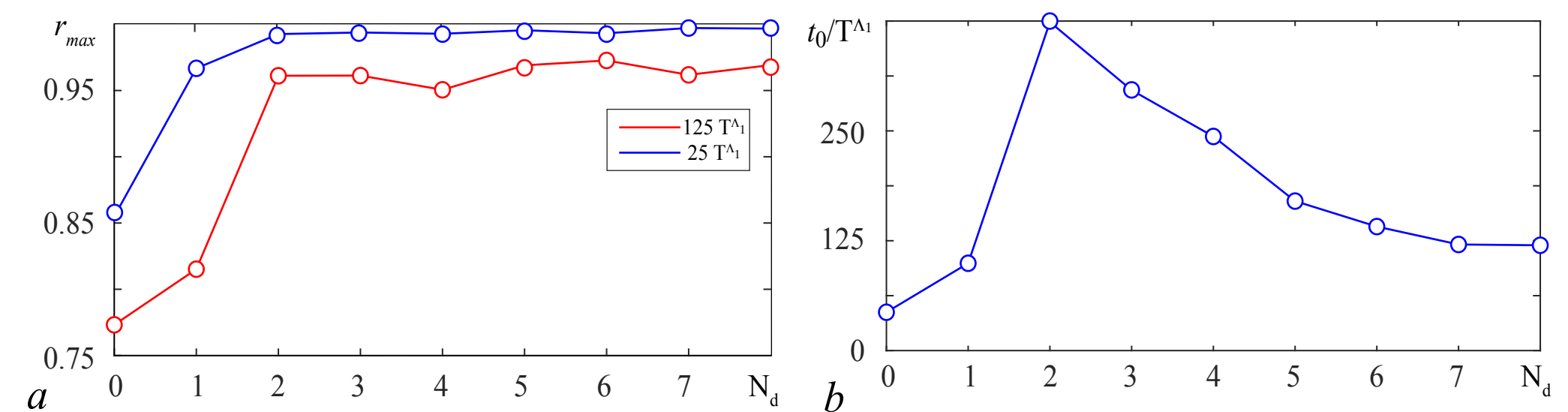


Fig. 4. (a) The maximal correlation  $r_{\max}$  between the actual and predicted macroscopic signals during 25 (black) and 125 (red) Lyapunov times  $T^{\Lambda_1}$  versus the number of the delayed coordinates  $N_d$ . (b) The dependence of the time interval  $t_0$  during which the correlation  $r > 0.8$  on the number of delay coordinates  $N_d$ .  $T^{\Lambda_1} = 1.25$  s.

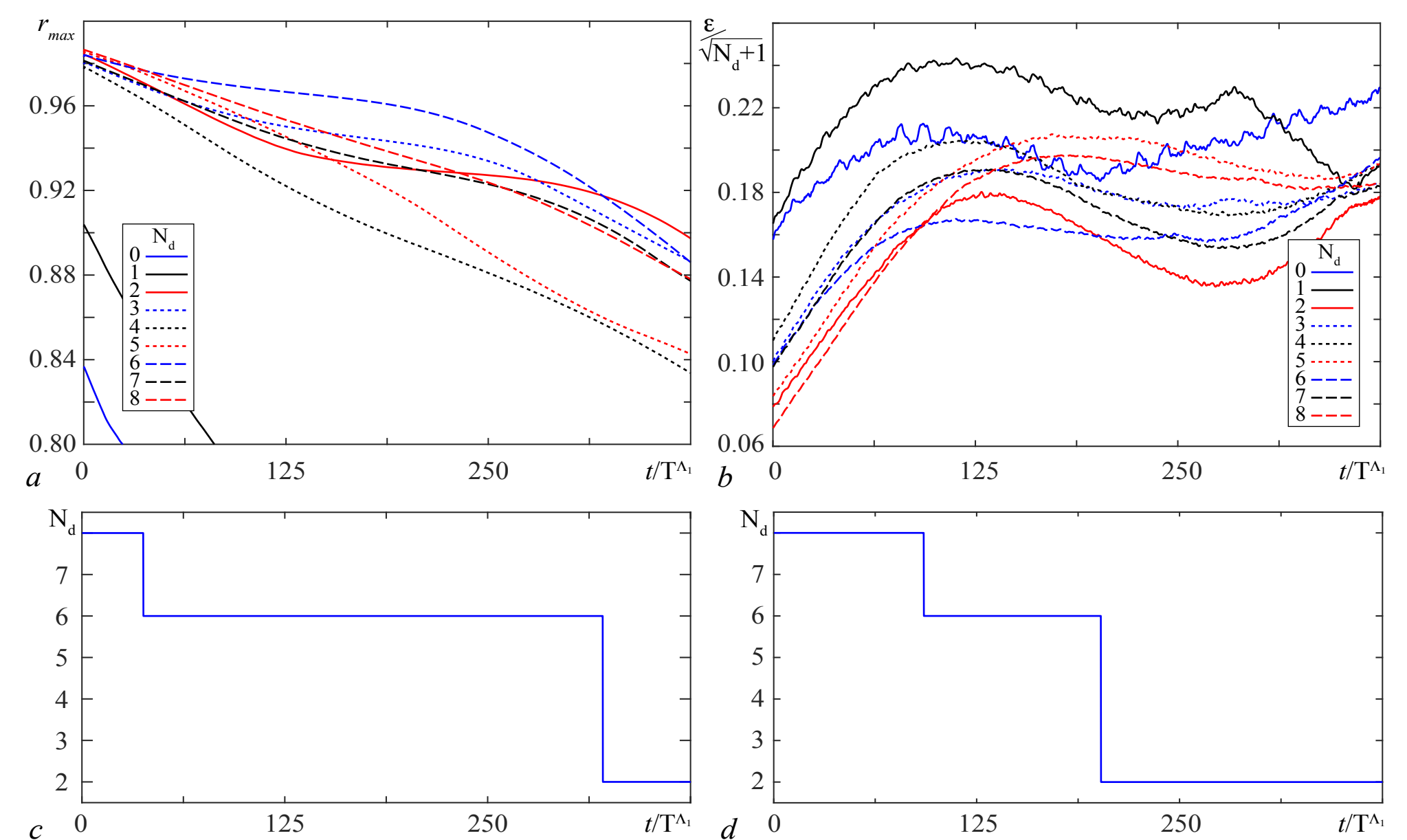


Fig. 5. (a) Time dependencies of (a) the maximal correlation  $r_{\max}$  and (b) the normalized error  $\epsilon = p/(N_d + 1)^{1/2}$  between the actual and predicted macroscopic signals and among all the considered parameters  $(D, R, \sigma_{\text{in}})$  for different number of delayed coordinates  $N_d$ . (c) The number of delay coordinates  $N_d$  for which correlation in Fig. (a) is maximal. (d) The number of delay coordinates  $N_d$  for which  $\epsilon = p/(N_d + 1)^{1/2}$  in Fig. (b) is minimal.

## Conclusions

- We investigated the capability of reservoir computing to predict the macroscopic signal generated by the adaptive network. Using Lyapunov analysis, we confirmed the chaotic nature of the generated macroscopic signal.
- We demonstrated that the reservoir trained on the raw macroscopic signal failed to predict it. To improve the prediction quality, we reconstructed the phase space by adding the delayed signals. Using these delayed signals as the reservoir input increased the accuracy of the prediction.
- We found that the correlation between the original and the predicted signal peaked for two delays and remained unchanged with a further increase in the delays.
- We found that the optimal number of delays depended on the prediction horizon: the long-term prediction required fewer input signals and vice-versa. The potential reason is that increasing the number of delays has a positive and negative impact: more delays give more information about the macroscopic signal to the reservoir but contribute to a faster increase in error during the iterative process.

## References

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