Dynamics of entanglement of two qubits interacting with two resonators

Mikhail M. Evseev, Eugene K. Bashkirov
Samara University

Model and its solution

Hamiltonian

$$\widehat{\mathcal{H}} = \hbar\omega_{qA}\hat{J}_A^z + \hbar\omega_{fA}\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\omega_{qB}\hat{J}_B^z + \hbar\omega_{fB}\left(\hat{b}^\dagger\hat{b} + \frac{1}{2}\right) +$$
$$+ \hbar\gamma_A\left(\hat{a}^N\hat{J}_A^+ + \left(\hat{a}^\dagger\right)^N\hat{J}_A^-\right) + \hbar\gamma_B\left(\hat{b}^N\hat{J}_B^+ + \left(\hat{b}^\dagger\right)^N\hat{J}_B^-\right) +$$
$$+ \hbar\varpi_A\mathcal{P}_A\left(\hat{a}^\dagger\hat{a}\right) + \hbar\varpi_B\mathcal{P}_B\left(\hat{b}^\dagger\hat{b}\right).$$

Initial state $|\Psi_0\rangle = \left(\cos(\theta)|+-\rangle + \sin(\theta)e^{I\phi_0}|-+\rangle\right)\otimes|n\,m\rangle.$

Time-dependent qubits density matrix

$$\hat{\rho}(\tau) = \begin{pmatrix} |C_1(\tau)|^2 + |C_2(\tau)|^2 & 0 & 0 & 0 \\ 0 & |C_3(\tau)|^2 + |C_4(\tau)|^2 & C_3(\tau)\bar{C}_5(\tau) & 0 \\ 0 & \bar{C}_3(\tau)C_5(\tau) & |C_5(\tau)|^2 + |C_6(\tau)|^2 & 0 \end{pmatrix}$$

$$C_1(\tau) = -2I\gamma_B\sqrt{(m-M+1)_M}\,e^{-I(\psi_A(n)+\psi_B(m-M))\tau}\left(\cos\left(\frac{\tau}{2}\mu_A(n)\right) + I\phi_A(n)\frac{\sin\left(\frac{\tau}{2}\mu_A(n)\right)}{\mu_A(n)}\right)\frac{\sin\left(\frac{\tau}{2}\mu_B(m-M)\right)}{\mu_B(m-M)}\cos(\theta),$$

$$C_2(\tau) = -2I\gamma_A\sqrt{(n-N+1)_N}\,e^{-I(\psi_A(n-N)+\psi_B(m))\tau}\left(\cos\left(\frac{\tau}{2}\mu_B(m)\right) + I\phi_B(m)\frac{\sin\left(\frac{\tau}{2}\mu_B(m)\right)}{\mu_B(m)}\right)\frac{\sin\left(\frac{\tau}{2}\mu_A(n-N)\right)}{\mu_A(n-N)}\sin(\theta)e^{I\phi_0},$$

$$C_3(\tau) = e^{-I(\psi_A(n)+\psi_B(m-M))\tau}\left(\cos\left(\frac{\tau}{2}\mu_A(n)\right) + I\phi_A(n)\frac{\sin\left(\frac{\tau}{2}\mu_A(n)\right)}{\mu_A(n)}\right)\left(\cos\left(\frac{\tau}{2}\mu_B(m-M)\right) - I\phi_B(m-M)\frac{\sin\left(\frac{\tau}{2}\mu_B(m-M)\right)}{\mu_B(m-M)}\right)\cos(\theta),$$

$$C_4(\tau) = -4\gamma_A\gamma_B\sqrt{(n-N+1)_N(m+1)_M}\,e^{-I(\psi_A(n-N)+\psi_B(m))\tau}\frac{\sin\left(\frac{\tau}{2}\mu_A(n-N)\right)\sin\left(\frac{\tau}{2}\mu_B(m)\right)}{\mu_A(n-N)\mu_B(m)}\sin(\theta)e^{I\phi_0},$$

$$C_5(\tau) = e^{-I(\psi_A(n-N)+\psi_B(m))\tau}\left(\cos\left(\frac{\tau}{2}\mu_A(n-N)\right) - I\phi_A(n-N)\frac{\sin\left(\frac{\tau}{2}\mu_A(n-N)\right)}{\mu_A(n-N)}\right)\left(\cos\left(\frac{\tau}{2}\mu_B(m)\right) + I\phi_B(m)\frac{\sin\left(\frac{\tau}{2}\mu_B(m)\right)}{\mu_B(m)}\right)\sin(\theta)e^{I\phi_0},$$

$$C_6(\tau) = -4\gamma_A\gamma_B\sqrt{(n+1)_N(m-M+1)_M}\,e^{-I(\psi_A(n)+\psi_B(m-M))\tau}\frac{\sin\left(\frac{\tau}{2}\mu_A(n)\right)\sin\left(\frac{\tau}{2}\mu_B(m-M)\right)}{\mu_A(n)\mu_B(m-M)}\cos(\theta),$$

$$C_7(\tau) = -2I\gamma_B\sqrt{(m+1)_M}\,e^{-I(\psi_A(n-N)+\psi_B(m))\tau}\left(\cos\left(\frac{\tau}{2}\mu_A(n-N)\right) - I\phi_A(n-N)\frac{\sin\left(\frac{\tau}{2}\mu_A(n-N)\right)}{\mu_A(n-N)}\right)\frac{\sin\left(\frac{\tau}{2}\mu_B(m)\right)}{\mu_B(m)}\sin(\theta)e^{I\phi_0},$$

$$C_8(\tau) = -2I\gamma_A\sqrt{(n+1)_N}\,e^{-I(\psi_A(n)+\psi_B(m-M))\tau}\left(\cos\left(\frac{\tau}{2}\mu_B(m-M)\right) - I\phi_B(m-M)\frac{\sin\left(\frac{\tau}{2}\mu_B(m-M)\right)}{\mu_B(m-M)}\right)\frac{\sin\left(\frac{\tau}{2}\mu_A(n)\right)}{\mu_A(n)}\cos(\theta),$$

$$\psi_{A(B)}(n) = \omega_{fA(B)}\left(n + \frac{N(M)+1}{2}\right) + \varpi_{A(B)}\frac{\mathcal{P}_{A(B)}(n+N(M)) + \mathcal{P}_{A(B)}(n)}{2},\quad \phi_{A(B)}(n) = \delta_{A(B)} + \varpi_{A(B)}\left(\mathcal{P}_{A(B)}(n+N(M)) - \mathcal{P}_{A(B)}(n)\right),$$

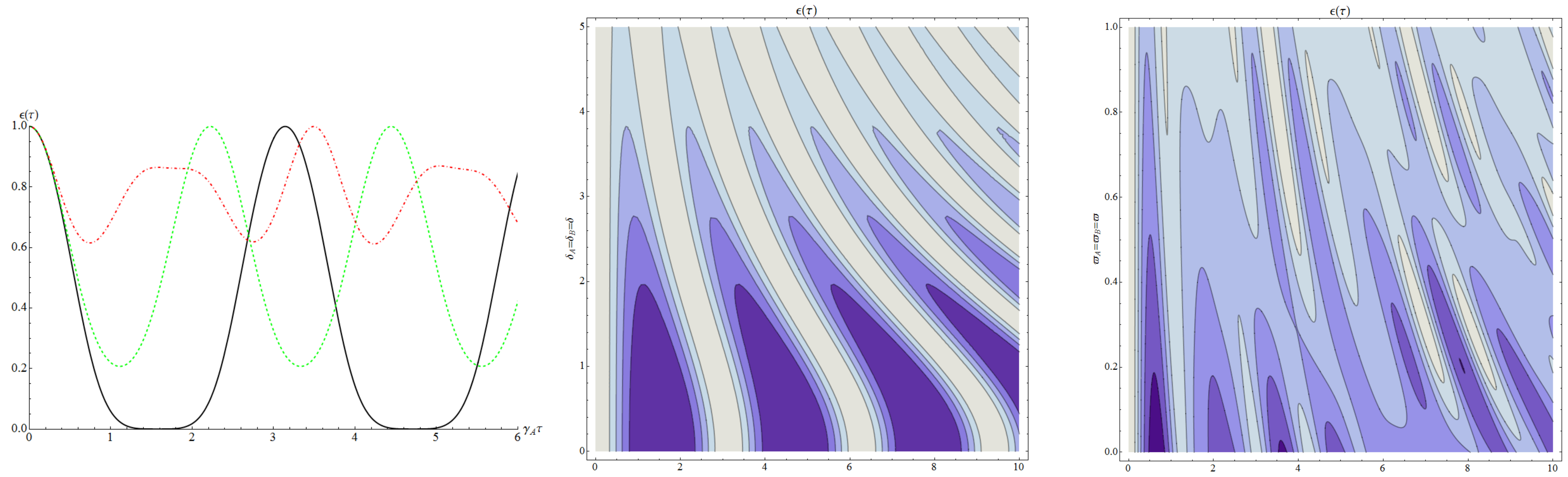
$$\mu_{A(B)}(n) = \sqrt{\phi_{A(B)}^2 + 4\gamma_{A(B)}^2(n+1)_{N(M)}},\qquad \delta_{A(B)} = N(M)\,\omega_{fA(B)} - \omega_{qA(B)},\quad (x)_N = x(x+1)\ldots(x+N-1).$$

Negativity and its numerical modeling

Negativity

$$\varepsilon(\tau) = \sqrt{\left(|C_1(\tau)|^2 + |C_2(\tau)|^2 - |C_7(\tau)|^2 - |C_8(\tau)|^2\right)^2 + 4|C_3(\tau)|^2|C_5(\tau)|^2} - \left(|C_1(\tau)|^2 + |C_2(\tau)|^2 + |C_7(\tau)|^2 + |C_8(\tau)|^2\right).$$

Numerical modeling



(a) (b) (c)

Fig. 1(a). The negativity as a function of a scaled time $\gamma_A\tau$ for $\gamma_B = \gamma_A = \gamma$, $n = m = 0$, $N = M = 1$, $\theta = \pi/4$, $\varpi_A = \varpi_B = 0$ and $\delta_A = \delta_B = 0$ (solid black), $\delta_A = \delta_B = 2\gamma$ (dashed green) and $\delta_A = 3\gamma$, $\delta_B = 5\gamma$ (dot dashed red).

Fig. 1(b). The negativity as a function of a scaled time $\gamma_A\tau$ and the detuning $\delta_A = \delta_B = \delta$ (in units of γ) for $\gamma_B = \gamma_A = \gamma$, $n = m = 0$, $N = M = 1$, $\theta = \pi/4$, $\varpi_A = \varpi_B = 0$. Color correspond value of negativity (white $\to \varepsilon(\tau) = 1$, black $\to \varepsilon(\tau) = 0$). Lines are plotted with step 0.2.

Fig. 1(c). The negativity as a function of a scaled time $\gamma_A\tau$ and the dispersive parts of the third–order nonlinearity of Kerr medium $\varpi_A = \varpi_B = \varpi$ (in units of γ) for $\gamma_B = \gamma_A = \gamma$, $n = m = 0$, $N = M = 1$, $\theta = \pi/4$, $\mathcal{P}_A(x) = \mathcal{P}_B(x) = x(x-1)$. Color correspond value of negativity (white $\to \varepsilon(\tau) = 1$, black $\to \varepsilon(\tau) = 0$). Lines are plotted with step 0.2.

Conclusion

We studied dynamics of two identical qubits nonresonantly interacting with two independent resonators in framework of two-atom Jaynes-Cummings model with nonlinear interaction with fields and examined the influence of the Kerr nonlinearity and other model parameters on the atom-atom entanglement. The atomic entanglement behavior for entangled initial qubit states and cavity Fock states was a subject of our investigation. We derived that for initial Fock states with 2 and more photons the Kerr nonlinearity enhances the amount of entanglement. We also found that the detunings between qubit transition frequencies and resonator field mode frequencies affect the frequency of oscillations of negativity but can't control the maximum degree of qubit-qubit entanglement. These results may be useful for quantum information processing based on the entanglement.

SAMARA NATIONAL RESEARCH UNIVERSITY
34, Moskovskoye shosse, Samara, Russia (www.ssau.ru)