Entanglement in Tavis-Cummings model with Kerr nonlinearity induced by a thermal noise

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Model and negativity calculations

Hamiltonian in the interaction picture

$$\hat{H} = \sum_{i=1}^{2} \hbar\gamma(\hat{\sigma}_i^{+}\hat{a} + \hat{\sigma}_i^{-}\hat{a}^{\dagger}) + \kappa\left(\hat{a}^{\dagger}\right)^2 \hat{a}^2,$$

Initial separable incoherent atomic states

$$|\Psi(0)\rangle_A = |+,-\rangle \quad \text{or} \quad |\Psi(0)\rangle_A = |+,+\rangle \qquad (1)$$

Initial separable coherent atomic states

$$|\Psi(0)\rangle_{A_1} = \cos\varphi_1 |+\rangle_1 + \sin\varphi_2 |-\rangle_1, \quad |\Psi(0)\rangle_{A_2} = \cos\varphi_2 |+\rangle_2 + \sin\varphi_2 |-\rangle_2 \qquad (2)$$

Initial entangled atomic states

$$|\Psi(0)\rangle_A = \cos\theta |+,-\rangle + \sin\theta |-,+\rangle \qquad (3)$$

Initial thermal cavity field state

$$\hat{\rho}_F(0) = \sum_n p_n |n\rangle\langle n|, \qquad p_n = \frac{\bar{n}^n}{(1+\bar{n})^{n+1}}, \qquad \bar{n} = \left(\exp[\hbar\omega / k_B T] - 1\right)^{-1},$$

Time-dependent density matrix

$$\hat{\rho}(t) = \sum_{i=1}^{4}\sum_{n=0}^{\infty} p_n |\Psi_{in}(t)\rangle\langle\Psi_{in}(t)|,$$

$$|\Psi_{in}\rangle = C_{i1n}|-,-,n+2\rangle + C_{i2n}|+,-,n+1\rangle + C_{i3n}|-,+,n+1\rangle + C_{i4n}|+,+,n\rangle$$

Partial transposition of the reduced atomic density matrix

for intial atomic states (1) and (3)

$$\hat{\rho}_A^{T_1}(t) = \left[Tr_F\rho(t)\right]^{T_1} = \begin{pmatrix} U(t) & 0 & 0 & H^*(t) \\ 0 & W(t) & 0 & 0 \\ 0 & 0 & V(t) & 0 \\ H(t) & 0 & 0 & R(t) \end{pmatrix}$$

Negativity

$$\varepsilon(t) = -2\sum_{i=1}^{4}\mu_i^{-} = \sqrt{\left(U(t) - R(t)\right)^2 + 4\left|H(t)\right|^2} - U(t) - R(t)$$

Simulations

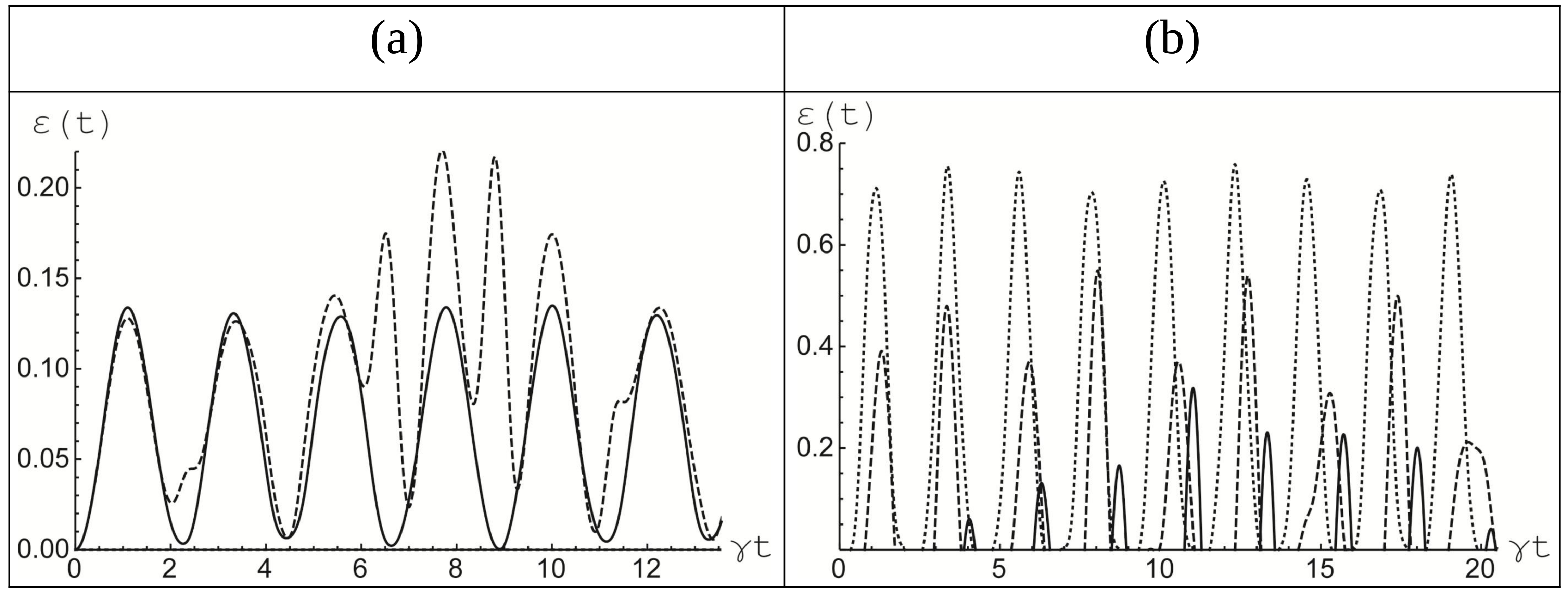

Fig. 1. The negativity as function of scaled time γt for initial separable atomic state $|+,-\rangle$ and $\chi = 0$ (solid), $\chi = 0.3$ (dashed) and $\chi = 1.5$ (dotted) (a) and atomic state $|+,+\rangle$ and $\chi = 0.5$ (solid), $\chi = 1.5$ (dashed). The mean photon number $\bar{n} = 0.5$. Parameter $\chi = \kappa / \hbar\gamma$.

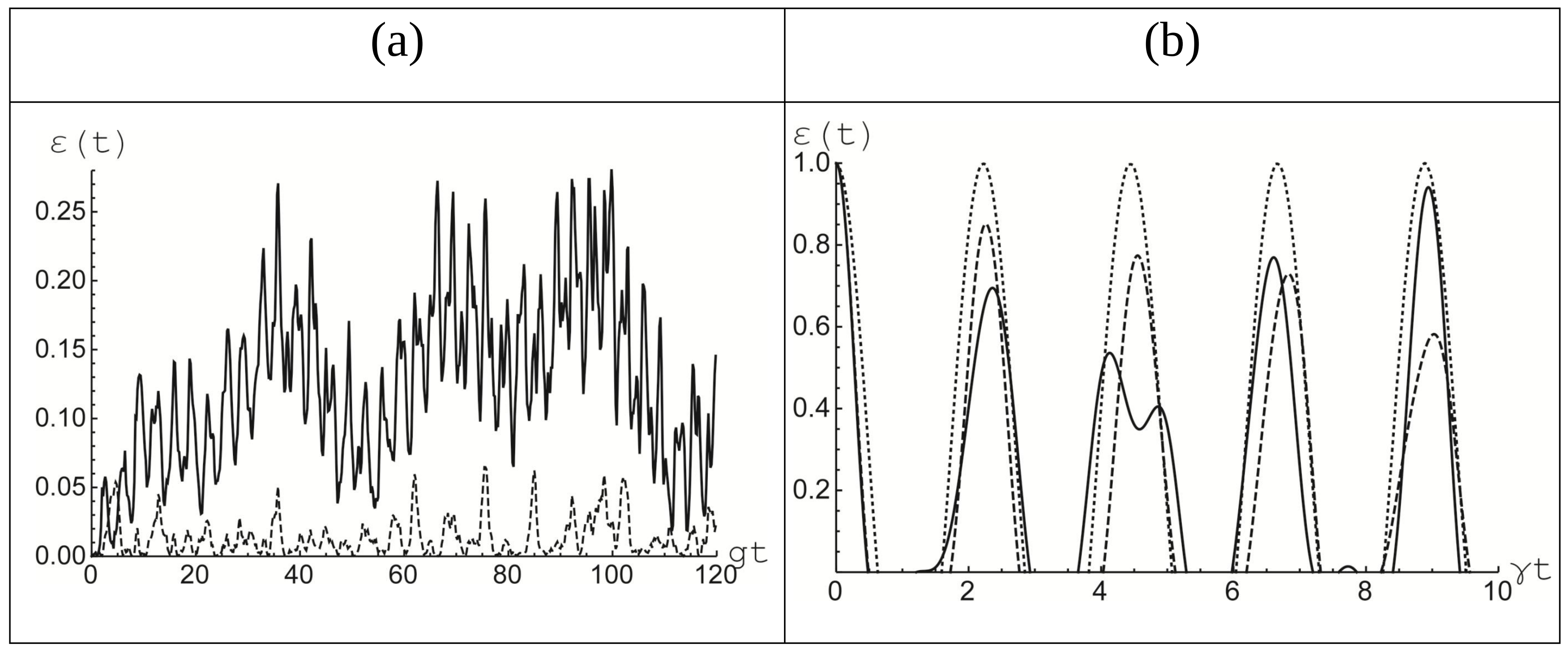

Fig. 2. The negativity as function of scaled time γt for initial atomic state $|+,-\rangle$ (solid) and coherent separable atomic state (2) with $\varphi_1 = \varphi_2 = \pi / 4$, parameter $\chi = 1$ (a) and entangled atomic state (3) with with $\theta = \pi / 4$, parameter $\chi = 0$ (solid), $\chi = 1$ (dashed) and $\chi = 15$ (dotted) (b). The mean photon number $\bar{n} = 0.1$. Parameter $\chi = \kappa / \hbar\gamma$.

Conclusion

We obtained that Kerr nonlinearity enhances the amount of atomic entanglement induced by a thermal field for separable noncoherent atomic states. The initial atomic coherence reduces the degree of atomic entanglement. For entangled initial atomic state the Kerr nonlinearity smoothes negativity amplitude fluctuations induced by a thermal noise, i.e. the stabilizations of entanglement takes place.

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