Entanglement in two-atom two-photon Tavis-Cummings model induced by a thermal field

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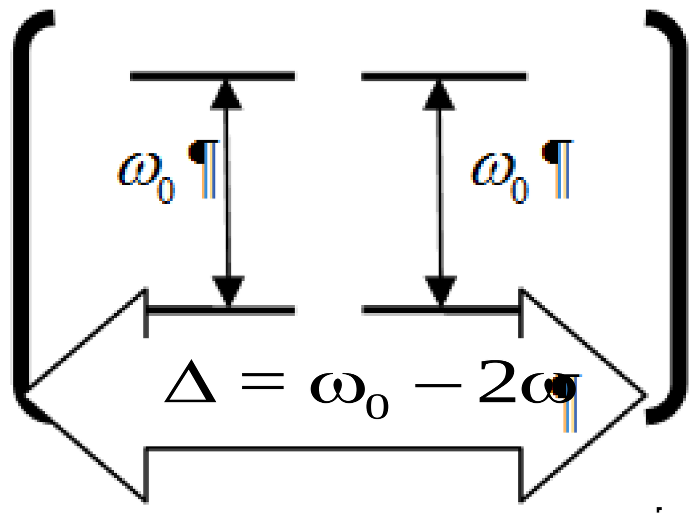
1 Model description

- The system consists of two identical qubits interacting with a one-mode thermal cavity mode with frequency ω via degenerate two-photon processess
- We consider the detuning between field and twice qubits frequencies
- The atom-field coupling is constant (thus we neglect the dependence of the spatial structure of the cavity mode)
- The qubits are asumed to prepared in separable or entangled initial states
- Assume that the total atom-cavity interaction time is considerably less than the cavity lifetime and that we can ignore the effects of cavity dissipation



Hamiltonian

$$H = (1/2)\hbar\Delta(\sigma_1^z + \sigma_2^z) + \hbar g \sum_{i=1}^{2} (\sigma_i^+ a^2 + a^{+2}\sigma_i^-),$$

Thermal initial field

$$\rho_F(0) = \sum_n p_n |n\rangle\langle n|, \quad p_n = \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \qquad \bar{n} = (\exp[\hbar\omega_i / k_B T] - 1]^{-1},$$

Atoms initial states

$$|\Psi(0)\rangle_{A_1A_2} = |+,-\rangle \qquad (1)$$

$$|\Psi(0)\rangle_{A_1A_2} = |+,+\rangle \qquad (2)$$

$$|\Psi(0)\rangle_{A_1A_2} = \cos\Theta|+,-\rangle + \sin\Theta|-,+\rangle \qquad (3)$$

Entanglement calculations

Peres-Horodetskii criterium (negativity) $\varepsilon = -2\sum_i \mu_i^-.$

Partially transported reduced qubits matrix

$$\rho_A^{T_1}(t) = \begin{pmatrix} U(t) & 0 & 0 & H(t)^* \\ 0 & V(t) & 0 & 0 \\ 0 & 0 & W(t) & 0 \\ H(t) & 0 & 0 & R(t) \end{pmatrix}.$$

Evident form of Peres-Horodetskii criterium (negativity)

$$\varepsilon(t) = \sqrt{(|R(t)| - |U(t)|)^2 + 4|H(t)|^2} - |R(t)| - |U(t)|$$

2 Dressed states

$$|\Phi_{in}\rangle = \xi_{in}(X_{i1n}|-,-,n+2\rangle + X_{i2n}|+,-,n+1\rangle + X_{i3n}|-,+,n+1\rangle + X_{i4n}|+,+,n\rangle), (i = 1,2,3,4),$$

$$X_{11,n} = 0, \quad X_{12,n} = -1, \quad X_{13,n} = 1, \quad X_{14,n} = 0,$$

$$X_{i1,n} = -\frac{2\sqrt{2+3n+n^2}\sqrt{12+7n+n^2}}{24+14n+2n^2-\delta\varepsilon_{in}-\varepsilon_{in}^2},$$

$$X_{i2,n} = X_{i3,n} = -\frac{\sqrt{2+3n+n^2}(\delta+\varepsilon_{in})}{24+14n+2n^2-\delta\varepsilon_{in}-\varepsilon_{in}^2},$$

$$X_{i4,n} = 1 \quad (i = 2,3,4), \quad \delta = \Delta / g$$

$$\varepsilon_{1n} = 0, \quad \varepsilon_{2n} = -\mathrm{Re}\left[\frac{2^{1/3}}{3\times X_n}\left(Y_n - X_n^2/2^{2/3}\right)\right],$$

$$\varepsilon_{3n} = \mathrm{Re}\left[\frac{1}{3\times 2^{2/3} X_n}\left(\left(1-i\sqrt{3}\right)Y_n - \left(1+i\sqrt{3}\right)X_n^2/2^{2/3}\right)\right],$$

$$,\left[\frac{1}{3\times 2^{2/3} X_n}\left(\left(1+i\sqrt{3}\right)Y_n - \left(1-i\sqrt{3}\right)X_n^2/2^{2/3}\right)\right],$$

$$X_n = \left(Z_n + \sqrt{Z_n^2 + 4Y_n^3}\right)^{1/3}, \quad \varepsilon_{4n} = \mathrm{Re}\,Y_n = -84-60n-12n^2-3\delta^2,$$

$$Z_n = -216\delta(5/2+n), \quad \varepsilon_{in} = E_{in}/\hbar g$$

3 Computer modelling

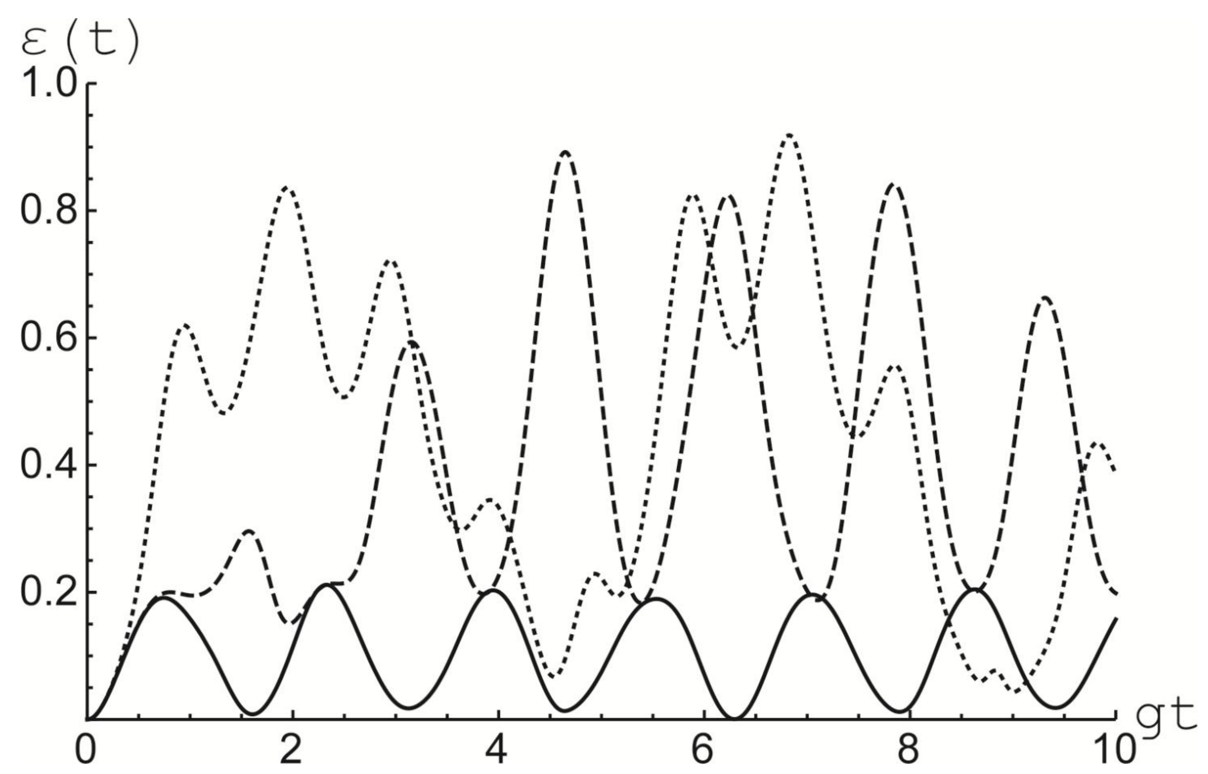


Fig. 1. Negativity vs gt for initial atomic state (1), $\delta = 0$ (solid), $\delta = 0,5$ (dashed) and $\delta = 5$ (dotted). Mean photon number $\bar{n} = 0,1$.

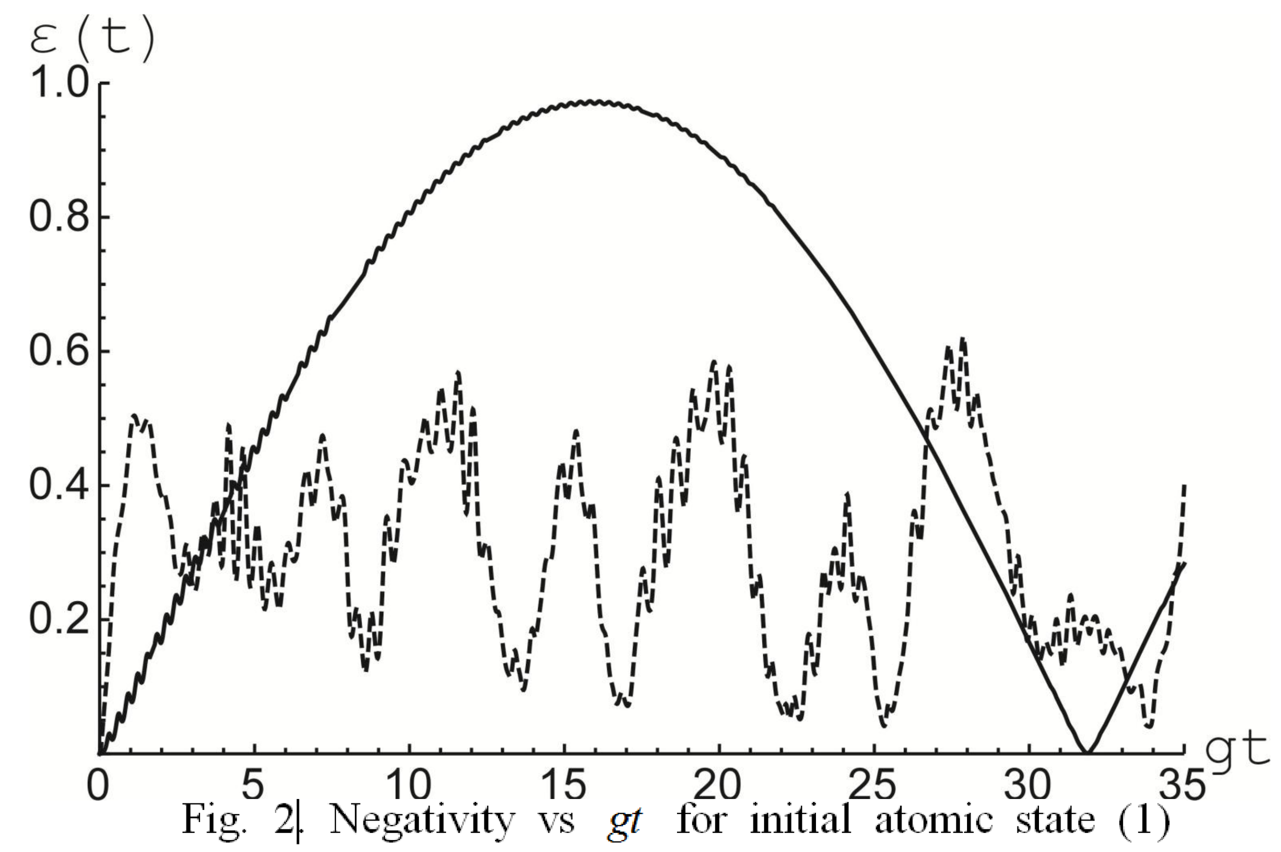


Fig. 2. Negativity vs gt for initial atomic state (1) one-photon interaction (a) and two-photon interaction (b), $\delta = 0,5$. Mean photon number $\bar{n} = 0,1$.

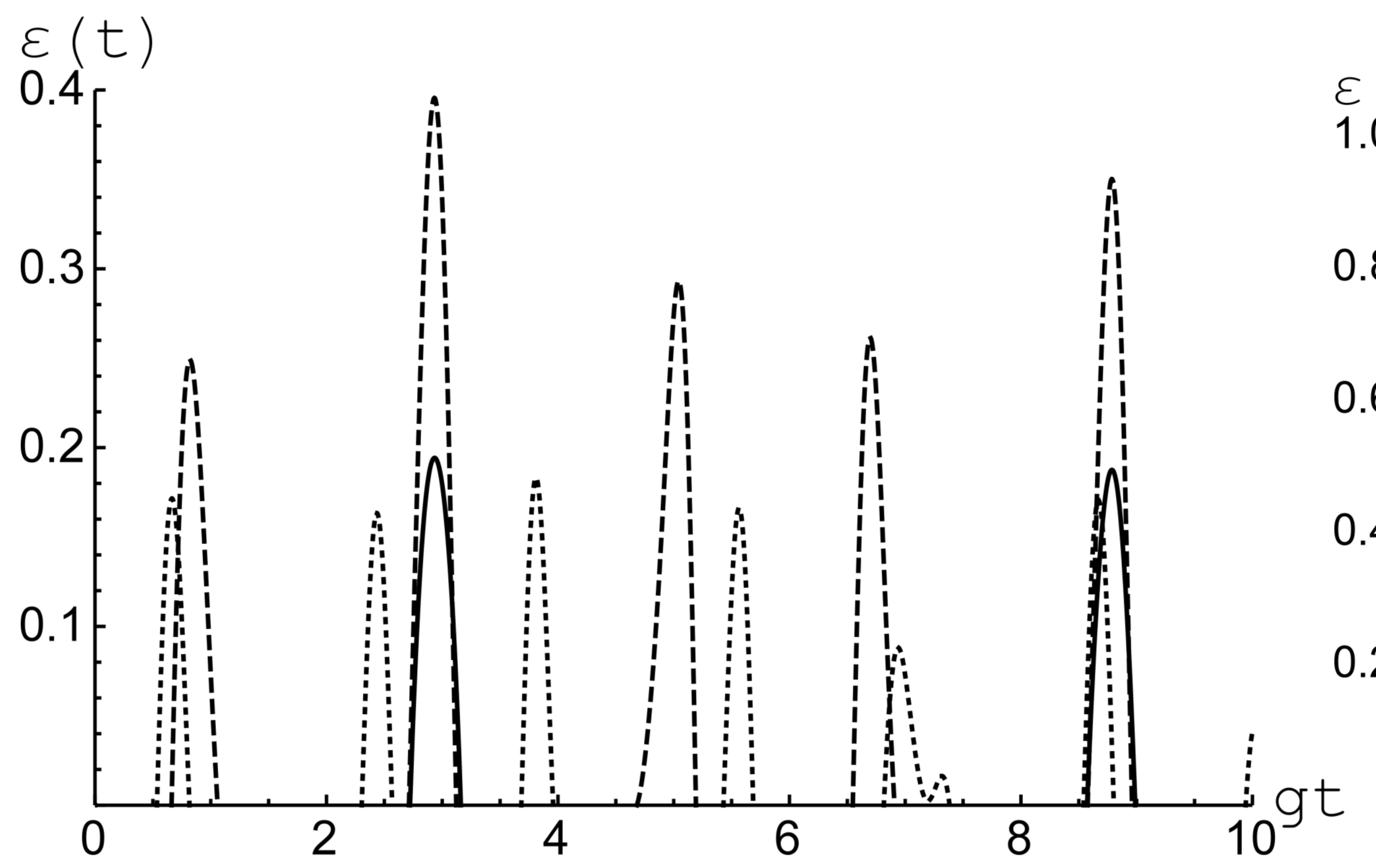


Fig. 3. Negativity vs gt for initial atomic state (2) and $\delta = 1$ (solid), $\delta = 3$ (dashed) and $\delta = 5$ (dotted). Mean photon number $\bar{n} = 0,1$

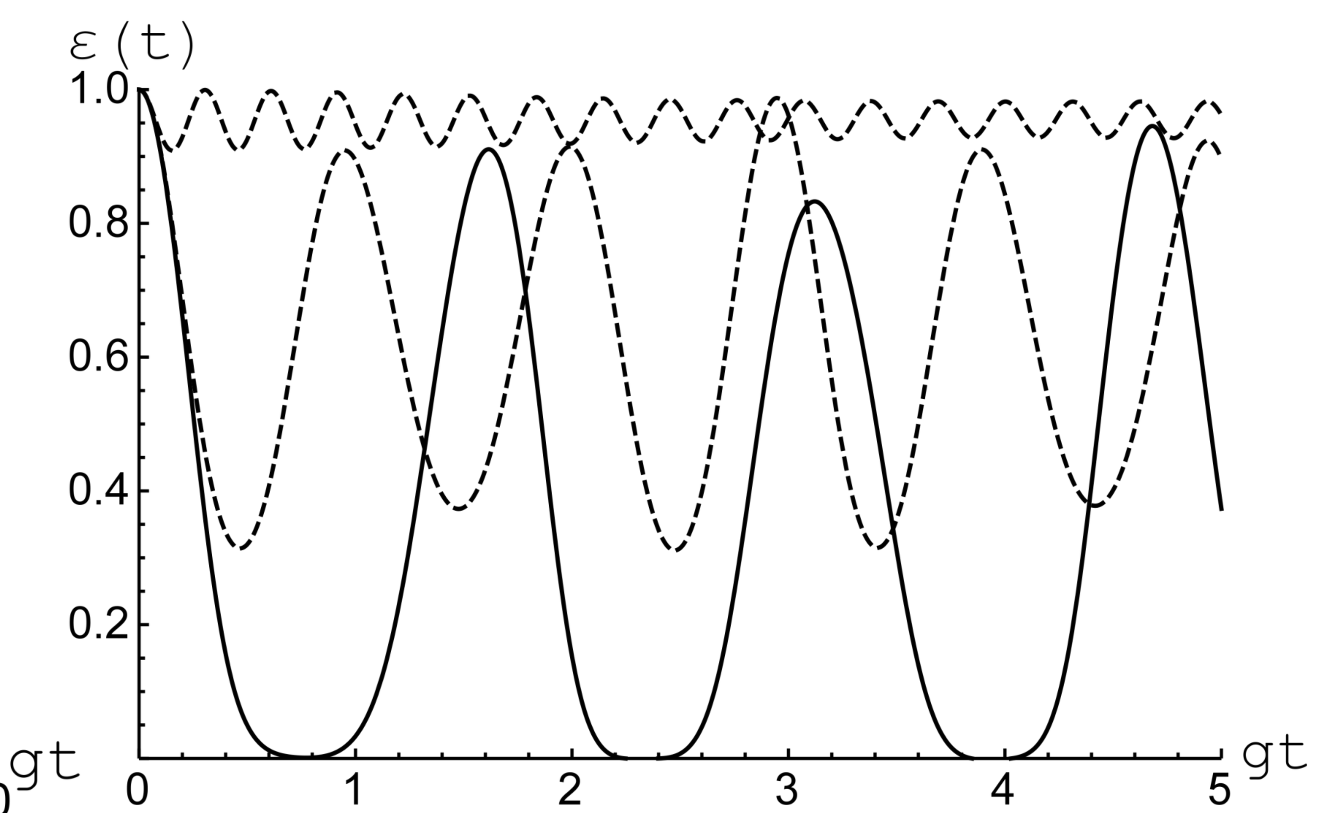


Fig. 4. Negativity vs gt for entangled initial atomic state (3) with $\theta = \pi/4$ and $\delta = 0$ (solid), $\delta = 5$ (dashed) and $\delta = 20$ (dotted). Mean photon number $\bar{n} = 0,1$